3. Find the critical point(s) for the function $f(x,y) = y^3 - 3yx^2 - 3y^2 - 3x^2 + 1$ and classify them using the second partials test. (15 points)

$f_y = 3y^2 - 3x^2 - 6y = 0$

$f_x = -6xy - 6x = 0$

$-6x(y+1) = 0$

$x = 0$ or $y = -1$

$x = 0 \quad 3y^2 - 6y = 0$

$3y(y-2) = 0$

$y = 0 \quad y = 2$

max $\boxed{(0,0) \quad (0,2)}$ Saddle

$y = -1$

$3(-1)^2 - 3x^2 - 6(-1) = 0$

$3 - 3x^2 + 6 = 0$

$9 - 3x^2 = 0$

$3x^2 = 9 \Rightarrow x^2 = 3 \quad x = \pm\sqrt{3}$

$\boxed{(\sqrt{3}, -1) \quad (-\sqrt{3}, -1)}$ both saddles

$f_{xx} = -6y - 6$

$f_{xy} = -6x$

$f_{yy} = 6y - 6$

$D: f_{xx} f_{yy} - (f_{xy})^2 =$

$f_{xx}(0,0) = -6$

$f_{xy}(0,0) = 0$

$f_{yy}(0,0) = -6$

36

$f_{xx}(0,2) = -18$

$f_{xy}(0,2) = 0$

$f_{yy}(0,2) = 6$

-108

$f_{xx}(\pm\sqrt{3}, -1) = 0$

$f_{xy}(\pm\sqrt{3}, -1) = \mp 6\sqrt{3}$

$f_{yy}(\pm\sqrt{3}, -1) = -12$

-108

4. Find the critical point(s) for the function $f(x,y) = y^2 - 2x^2 - 4z^2 - 6xz + 5z - 11$. (7 points)

$f_x = -4x - 6z = 0 \qquad -4x = 6z \qquad z = -\frac{4}{6}x = -\frac{2}{3}x$

$f_y = 2y = 0 \qquad y = 0$

$f_z = -8z - 6x + 5 = 0$

$+8(+\frac{2}{3}x) - 6x + 5 = 0$

$\frac{16}{3}x - 6x + 5 = 0$

$-\frac{2}{3}x = -5$

$x = \frac{15}{2}$

$z = -\frac{2}{3}\left(\frac{15}{2}\right) = -5$

$\boxed{\left(\frac{15}{2}, 0, -5\right)}$

7. <u>Maximize</u> $f(x,y,z) = xyz$ subject to the constraints $x+y+z=32,\ and\ x-y+z=0$. (15 points)

$F(x,y,z,\lambda,\mu) = xyz - \lambda(x+y+z-32) - \mu(x-y+z)$

$F_x = yz - \lambda - \mu = 0$ $\qquad yz = \lambda + \mu$

$F_y = xz - \lambda + \mu = 0$

$F_z = xy - \lambda - \mu = 0$ $\qquad xy = \lambda + \mu$

$yz - xy = 0$

$y(z-x) = 0$

$y = 0$ or $x = z$

$F_\lambda \Rightarrow x+y+z=32 \Rightarrow 2x+y=32$

$F_\mu \Rightarrow x-y+z=0 \Rightarrow 2x-y=0 \Rightarrow y=2x \qquad y=16$

$4x = 32$

$x = 8$

$z = 8$

$\boxed{(8,16,8)}$

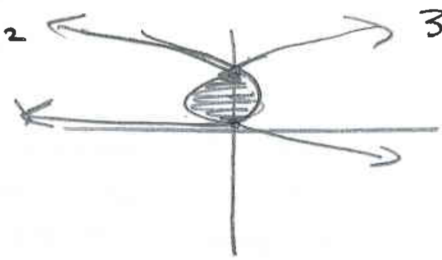
8. Sketch the region in the plane for the following integrals. (7 points each)

a. $\int_0^2 \int_{3y^2-6y}^{2y-y^2} 3y\,dxdy$

$y=0$
$y=2$

$x = 2y - y^2$

$x = 3y^2 - 6y$



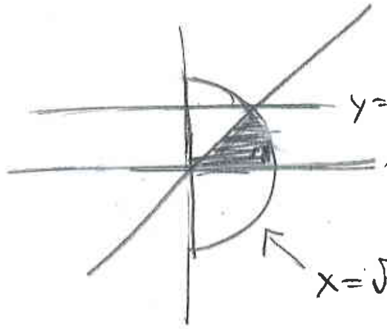
b. $\int_0^2 \int_y^{\sqrt{8-y^2}} \sqrt{x^2+y^2}\,dxdy$

$x = \sqrt{8-y^2}$

$x^2+y^2=8$

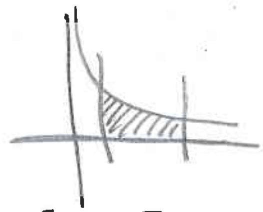
$x=y$

$y=0$
$y=2$



10. Find the center of mass for the lamina over the given region with the given density: $xy = 4, x = 1, x = 4, \rho = kx^2$. (20 points) $y = 0$

$y = \frac{4}{x}$



$$M = \int_1^4 \int_0^{4/x} kx^2 \, dy\, dx = \int_1^4 kx^2 y \Big|_0^{4/x} dx =$$

$$\int_1^4 k\cancel{x^2} \cdot \frac{4}{\cancel{x}} \, dx = \int_1^4 4kx \, dx = 2kx^2 \Big|_1^4 = 2k[16-1] = 30k$$

$$M_x = \int_1^4 \int_0^{4/x} kx^2 y \, dy\, dy = \int_1^4 \tfrac{1}{2}kx^2y^2 \Big|_0^{4/x} dx = \int_1^4 \tfrac{1}{2}k\cancel{x^2}\left(\frac{4}{\cancel{x}}\right)^2 dx = \int_1^4 8k \, dx =$$

$$8kx \Big|_1^4 = 8k(4-1) = 24k$$

$$M_y = \int_1^4 \int_0^{4/x} kx^3 \, dy\, dx = \int_1^4 kx^3 y \Big|_0^{4/x} dx = \int_1^4 k\cancel{x^3}^{\,2}\left(\frac{4}{\cancel{x}}\right) dx = \int_1^4 4kx^2 \, dx$$

$$= \frac{4}{3}kx^3 \Big|_1^4 = \frac{4}{3}k(64-1) = \frac{4}{\cancel{3}}k(\cancel{63}^{\,21}) = 84k$$

$$\bar{x} = \frac{M_y}{M} = \frac{84\cancel{k}}{30\cancel{k}} = \frac{14}{5} \qquad \bar{y} = \frac{M_x}{M} = \frac{24\cancel{k}}{30\cancel{k}} = \frac{4}{5} \qquad \boxed{(\bar{x}, \bar{y}) = \left(\frac{14}{5}, \frac{4}{5}\right)}$$

11. Find the area of the surface of the cone $z = 2\sqrt{x^2 + y^2}$ inside the cylinder $x^2 + y^2 = 4$. (15 points)

$$z = 2(x^2+y^2)^{1/2}$$

$$z_x = \cancel{2 \cdot \tfrac{1}{2}(x^2+y^2)^{-1/2} \cdot 2x}$$

$$\frac{2x}{\sqrt{x^2+y^2}}$$

$$z_y = \frac{2y}{\sqrt{x^2+y^2}}$$

$$\iint_R \sqrt{1 + \left(\frac{2x}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{2y}{\sqrt{x^2+y^2}}\right)^2} \, dA =$$

$$\iint_R \sqrt{1 + \frac{4x^2 + 4y^2}{x^2+y^2}} \, dA = \iint_R \sqrt{1 + \frac{4\cancel{(x^2+y^2)}}{\cancel{x^2+y^2}}} \, dA$$

$$= \iint_R \sqrt{1+4} \, dA = \iint_R \sqrt{5} \, dA$$

in polar $R = \; r \le 2$

$$\int_0^{2\pi} \int_0^2 \sqrt{5}\, r \, dr\, d\theta = \int_0^{2\pi} \frac{\sqrt{5}}{2} r^2 \Big|_0^2 d\theta = \int_0^{2\pi} \frac{\sqrt{5}}{\cancel{2}} \cancel{4}^{\,2} \, d\theta =$$

$$\int_0^{2\pi} 2\sqrt{5} \, d\theta = 2\sqrt{5}(2\pi - 0) = \boxed{4\sqrt{5}\,\pi}$$