Name KEY

Math 254, Quiz #12, Winter 2012

Instructions: Show all work. Use exact answers unless directed otherwise (with the exception of some application problems). Problems with answers only will rarely receive full credit. Be sure to read each problem carefully and complete all parts.

1. Find the center of mass of the volume with density function $\rho(x,y,z)=k(4-z)$ over the region bounded by $y=\sqrt{9-x^2}, z=y, z=0$. (7 points)

$x^2+y^2=9$, $y\geq 0$, cylindrical, $y = r\sin\theta$

$$M=\int_0^{\pi}\int_0^3\int_0^{r\sin\theta} k(4-z)\,r\,dz\,dr\,d\theta = k\int_0^{\pi}\int_0^3\left(4z-\frac{z^2}{2}\right)\Big|_0^{r\sin\theta} r\,dr\,d\theta =$$

$$k\int_0^{\pi}\int_0^3 4r^2\sin\theta-\frac{1}{2}r^3\sin^2\theta\,dr\,d\theta = k\int_0^{\pi}\frac{4}{3}r^3\sin\theta-\frac{1}{8}r^4\sin^2\theta\Big|_0^3 d\theta =$$

$$k\int_0^{\pi} 36\sin\theta-\frac{81}{8}\sin^2\theta\,d\theta = k(56.09568719)$$

$$M_{xy}=k\int_0^{\pi}\int_0^3\int_0^{r\sin\theta}(4z-z^2)\,r\,dz\,dr\,d\theta = k\int_0^{\pi}\int_0^3 2z^2-\frac{1}{3}z^3\Big|_0^{r\sin\theta} r\,dr\,d\theta = k\int_0^{\pi}\int_0^3 2r^3\sin^2\theta-\frac{1}{3}r^4\sin^3\theta\,dr\,d\theta$$

$$= k\int_0^{\pi}\frac{1}{2}r^4\sin^2\theta-\frac{1}{15}r^5\sin^3\theta\Big|_0^3 d\theta = k\int_0^{\pi}\frac{81}{2}\sin^2\theta-\frac{81}{5}\sin^3\theta\,d\theta = k(42.01725124)$$

$$M_{xz}=k\int_0^{\pi}\int_0^3\int_0^{r\sin\theta}(4-z)\,r^2\sin\theta\,dz\,dr\,d\theta = k\int_0^{\pi}\int_0^3 4z-\frac{1}{2}z^2\Big|_0^{r\sin\theta} r^2\sin\theta\,dr\,d\theta =$$

$$= k\int_0^{\pi}\int_0^3 4r^3\sin^2\theta-\frac{1}{2}r^4\sin^3\theta\,dr\,d\theta = k\int_0^{\pi} r^4\sin^2\theta-\frac{1}{10}r^5\sin^3\theta\Big|_0^3 d\theta = k\int_0^{\pi}81\sin^2\theta-\frac{243}{10}\sin^3\theta\,d\theta =$$

$$= k(94.83450247)$$

2. Integrate. (4 points each)

a. $$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\cos\varphi}\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = \int_0^{2\pi}\int_0^{\pi/4}\frac{\rho^3}{3}\Big|_0^{\cos\varphi}\sin\varphi\,d\varphi\,d\theta = \frac{1}{3}\int_0^{2\pi}\int_0^{\pi/4}\cos^3\varphi\sin\varphi\,d\varphi\,d\theta =$$

$$\frac{1}{3}\int_0^{2\pi}-\frac{1}{4}\cos^4\varphi\Big|_0^{\pi/4}d\theta = \frac{-1}{12}\int_0^{2\pi}\left(\frac{1}{4}-1\right)d\theta = \frac{1}{12}\left(\frac{3}{4}\right)2\pi = \boxed{\frac{\pi}{8}}$$

b. $$\int_0^{\pi/2}\int_0^{2\cos^2\theta}\int_0^{4-r^2} r\sin\theta\,dz\,dr\,d\theta = \int_0^{\pi/2}\int_0^{2\cos^2\theta}(4r-r^3)\sin\theta\,dr\,d\theta =$$

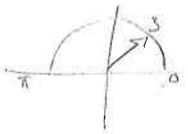
$$\int_0^{\pi/2} 2r^2-\frac{1}{4}r^4\Big|_0^{2\cos^2\theta}\sin\theta\,d\theta = \int_0^{\pi/2}(8\cos^4\theta-4\cos^8\theta)\sin\theta\,d\theta$$

$u=\cos\theta$

$-du=\sin\theta$

$$-\frac{8}{5}\cos^5\theta+\frac{4}{9}\cos^9\theta\Big|_0^{\pi/2} =$$

$$-\frac{8}{5}(0)+\frac{4}{9}(0)+\frac{8}{5}(1)-\frac{4}{9}(1) = \boxed{\frac{52}{45}}$$



$$M_{yz} = k\int_0^{\pi}\int_0^{3}\int_0^{r\sin\theta} (4-z)\, r^2\cos\theta \, dz\, dr\, d\theta = k\int_0^{\pi}\int_0^{3} \left(4z - \tfrac{1}{2}z^2\right)\Big|_0^{r\sin\theta} r^2\cos\theta \, dr\, d\theta =$$

$$k\int_0^{\pi}\int_0^{3} 4r^3\sin\theta\cos\theta - \tfrac{1}{2}r^4\sin^2\theta\cos\theta \, dr\, d\theta =$$

$$k\int_0^{\pi} r^4\sin\theta\cos\theta - \tfrac{1}{10}r^5\sin^2\theta\cos\theta \Big|_0^{3} d\theta = k\int_0^{\pi} 81\sin\theta\cos\theta - \tfrac{243}{10}\sin^2\theta\cos\theta \, d\theta =$$

$$= k(0)$$

$$\bar{x} = \frac{M_{yz}}{M} = 0$$

$$\bar{y} = \frac{M_{xz}}{M} = 1.69059\ldots$$

$$\bar{z} = \frac{M_{xy}}{M} = .749028\ldots$$

$$(\bar{x}, \bar{y}, \bar{z}) \approx \boxed{(0,\ 1.7,\ .75)}$$