Study Guide #2 (254)

1. a) $w = \cos(x-y)$ $\quad x = t^2 \quad y = 1$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dt}$$

$$\frac{\partial w}{\partial x} = -\sin(x-y) \qquad \frac{\partial w}{\partial y} = +\sin(x-y) \qquad \frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = 0$$

$$\frac{dw}{dt} = -\sin(x-y)(2t) + \underbrace{\sin(x-y)\cdot 0}_{\to 0} = -\sin(t^2-1)(2t) = -2t\sin(t^2-1)$$

Check:

$$w(t) = \cos(t^2-1) \qquad \frac{dw}{dt} = -\sin(t^2-1)(2t) = -2t\sin(t^2-1) \checkmark$$

b) $w = \sqrt{4-2x^2-2y^2} = (4-2x^2-2y^2)^{1/2}$ $\quad x = r\cos\theta \quad y = r\sin\theta$

$$\frac{\partial w}{\partial x} = \frac{1}{\cancel{2}}(4-2x^2-2y^2)^{-1/2}\cdot -\cancel{4}^{2}x = -2x(4-2x^2-2y^2)^{-1/2} = \frac{-2x}{\sqrt{4-2x^2-2y^2}}$$

$$\frac{\partial w}{\partial y} = \frac{1}{2}(4-2x^2-2y^2)^{-1/2}(4y) = -2y(4-2x^2-2y^2)^{-1/2} = \frac{-2y}{\sqrt{4-2x^2-2y^2}}$$

$$\frac{\partial x}{\partial r} = \cos\theta \qquad \frac{\partial x}{\partial \theta} = -r\sin\theta \qquad \frac{\partial y}{\partial r} = \sin\theta \qquad \frac{\partial y}{\partial \theta} = r\cos\theta$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r} = \frac{-2x}{\sqrt{4-2x^2-2y^2}}\cdot\cos\theta + \frac{-2y}{\sqrt{4-2x^2-2y^2}}\sin\theta$$

$$= \frac{-2r\cos^2\theta - 2r\sin^2\theta}{\sqrt{4-2r^2}} = \frac{-2r(\underbrace{\cos^2\theta+\sin^2\theta}_{=1})}{\sqrt{4-2r^2}} = \frac{-2r}{\sqrt{4-2r^2}}$$

$$\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x}\,\frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial \theta} = \frac{-2x}{\sqrt{4-2x^2-2y^2}}\cdot -r\sin\theta + \frac{-2y}{\sqrt{4-2x^2-2y^2}}\,r\cos\theta$$

$$= \frac{2r^2\cos\theta\sin\theta - 2r^2\sin\theta\cos\theta}{\sqrt{4-2r^2}} = 0$$

Check

$$w = \sqrt{4-2x^2-2y^2} = \sqrt{4-2r^2} = (4-2r^2)^{1/2}$$

$$\frac{\partial w}{\partial r} = \frac{1}{2}(4-2r^2)^{-1/2} \cdot -4r = \frac{-2r}{\sqrt{4-2r^2}} \checkmark$$

$$\frac{\partial w}{\partial \theta} = 0$$ since there is no θ in the equation ✓

2. a) long way

$$z = e^x \sin(y+z)$$

$$z_x = e^x \sin(y+z) + e^x \cos(y+z) \cdot z_x$$

$$z_x - e^x \cos(y+z) z_x = e^x \sin(y+z)$$

$$\frac{z_x(1-e^x\cos(y+z))}{1-e^x\cos(y+z)} = \frac{e^x \sin(y+z)}{1-e^x\cos(y+z)}$$

$$z_x = \frac{e^x\sin(y+z)}{1-e^x\cos(y+z)}$$

$$z_y = e^x \cos(y+z)(1+z_y)$$

$$z_y = e^x\cos(y+z) + z_y e^x \cos(y+z)$$

$$z_y - z_y e^x \cos(y+z) = e^x \cos(y+z)$$

$$\frac{z_y(1-e^x\cos(y+z))}{1-e^x\cos(y+z)} = \frac{e^x\cos(y+z)}{1-e^x\cos(y+z)} \Rightarrow z_y = \frac{e^x\cos(y+z)}{1-e^x\cos(y+z)}$$

quick way:

$$F(x,y,z) = e^x\sin(y+z) - z$$

$$F_x = e^x \sin(y+z)$$

$$F_y = e^x\cos(y+z)$$

$$F_z = e^x\cos(y+z) - 1$$

$$z_x = -\frac{F_x}{F_z} = -\frac{e^x\sin(y+z)}{e^x\cos(y+z)-1} = \frac{e^x\sin(y+z)}{1-e^x\cos(y+z)}$$

$$z_y = -\frac{F_y}{F_z} = -\frac{e^x\cos(y+z)}{e^x\cos(y+z)-1} = \frac{e^x\cos(y+z)}{1-e^x\cos(y+z)}$$

b) long way

$$x^2 + 2xyz - y^2 - 3x + 7y - z^3 + 10 = 0 = F(x,y,z)$$

$$2x + 2yz + 2xyz_x - 3 - 3z^2 z_x = 0$$

$$2xyz_x - 3z^2 z_x = 3 - 2x - 2yz$$

$$\frac{z_x(2xy - 3z^2)}{2xy - 3z^2} = \frac{3 - 2x - 2yz}{2xy - 3z^2} \Rightarrow z_x = \frac{3 - 2x - 2yz}{2xy - 3z^2}$$

$$2xz + 2xyz_y - 2y + 7 - 3z^2 z_y = 0$$

$$2xyz_y - 3z^2 z_y = 2y - 7 - 2xz$$

$$\frac{z_y(2xy - 3z^2)}{2xy - 3z^2} = \frac{2y - 7 - 2xz}{2xy - 3z^2} \Rightarrow z_y = \frac{2y - 7 - 2xz}{2xy - 3z^2}$$

quick way:

$F_x = 2x + 2yz - 3$

$F_y = 2xz - 2y + 7$

$F_z = 2xy - 3z^2$

$$z_x = -\frac{F_x}{F_z} = -\frac{2x + 2yz - 3}{2xy - 3z^2} = \frac{3 - 2x - 2yz}{2xy - 3z^2} \checkmark$$

$$z_y = -\frac{F_y}{F_z} = -\frac{2xz - 2y + 7}{2xy - 3z^2} = \frac{2y - 7 - 2xz}{2xy - 3z^2} \checkmark$$

3. a) $f(x,y) = x^2 - y^2$

$\nabla f = 2x\hat{\imath} - 2y\hat{\jmath}$ $\quad \nabla f(3,4) = 6\hat{\imath} - 8\hat{\jmath}$

$$\nabla f(3,4) \cdot \vec{v} = (6\hat{\imath} - 8\hat{\jmath}) \cdot \left(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right) = \frac{6}{\sqrt{2}} - \frac{8}{\sqrt{2}} = \boxed{\frac{-2}{\sqrt{2}}}$$

b. $g(x,y) = \arcsin(xy)$

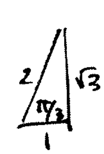
$$\nabla g = \frac{y}{\sqrt{1 - x^2y^2}}\hat{\imath} + \frac{x}{\sqrt{1 - x^2y^2}}\hat{\jmath} \qquad \nabla g \cdot \vec{u} = \hat{\jmath} \cdot \left(\frac{1}{\sqrt{26}}\hat{\imath} + \frac{5}{\sqrt{26}}\hat{\jmath}\right) =$$

$$\nabla g(1,0) = 0\hat{\imath} + \frac{1}{\sqrt{1-0}}\hat{\jmath} = \hat{\jmath} \qquad \boxed{\frac{5}{\sqrt{26}}}$$

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{26}}\hat{\imath} + \frac{5}{\sqrt{26}}\hat{\jmath}$$

$$\|\vec{v}\| = \sqrt{1 + 25} = \sqrt{26}$$

4. $h(x,y) = y\cos(x-y)$



$\nabla h = -y\sin(x-y)\,\hat{\imath} + [\cos(x-y) + y\sin(x-y)]\,\hat{\jmath}$

$\nabla h(0,\pi/3) = +\pi/3\sin(+\pi/3)\,\hat{\imath} + [\cos(+\pi/3) + \pi/3\sin(-\pi/3)]\,\hat{\jmath}$

$\frac{\pi}{3}\cdot\frac{1}{2}\,\hat{\imath} + [\frac{\sqrt{3}}{2} + \frac{\pi}{3}\cdot -\frac{1}{2}]\,\hat{\jmath} = \frac{\pi}{6}\,\hat{\imath} + (\frac{\sqrt{3}}{2} - \frac{\pi}{6})\,\hat{\jmath}$

5. $g(x,y) = ye^{-x^2}$

$\nabla g = -2xye^{-x^2}\,\hat{\imath} + e^{-x^2}\,\hat{\jmath}$ $\quad$ $\nabla g(0,5) = 0\,\hat{\imath} + e^0\,\hat{\jmath} = \hat{\jmath}$

$\|\nabla g\| = \sqrt{4x^2y^2e^{-2x^2} + e^{-2x^2}} = e^{-x^2}\sqrt{4x^2y^2+1} = \frac{\sqrt{4x^2y^2+1}}{e^{x^2}}$

6. ∇w for $w = x^2y^2z^2$

$\nabla w = 2xy^2z^2\,\hat{\imath} + 2x^2yz^2\,\hat{\jmath} + 2x^2y^2z\,\hat{k}$

$\nabla w(2,1,1) = 2(2)(1)^2(1)^2\,\hat{\imath} + 2(2)^2(1)(1)^2\,\hat{\jmath} + 2(2)^2(1)^2(1)\,\hat{k} = 4\hat{\imath} + 8\hat{\jmath} + 8\hat{k}$

7. $x^2+y^2+z^2 = 11 \Rightarrow F(x,y,z) = x^2+y^2+z^2-11 = 0$

$\nabla F = 2x\,\hat{\imath} + 2y\,\hat{\jmath} + 2z\,\hat{k}$

$\|\nabla F\| = \sqrt{4x^2+4y^2+4z^2} = 2\sqrt{x^2+y^2+z^2}$

$\hat{n} = \frac{\cancel{2}x\,\hat{\imath} + \cancel{2}y\,\hat{\jmath} + \cancel{2}z\,\hat{k}}{\cancel{2}\sqrt{x^2+y^2+z^2}} = \frac{x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}}{\sqrt{x^2+y^2+z^2}}$

8. $z = \sqrt{x^2+y^2}$

$F(x,y,z) = (x^2+y^2)^{1/2} - z$

$\nabla F = \frac{x}{\sqrt{x^2+y^2}}\,\hat{\imath} + \frac{y}{\sqrt{x^2+y^2}}\,\hat{\jmath} - 1\,\hat{k}$ $\quad$ $\nabla F(3,4,5) = \frac{3}{5}\,\hat{\imath} + \frac{4}{5}\,\hat{\jmath} - 1\,\hat{k}$

$\frac{3}{5}(x-3) + \frac{4}{5}(y-4) - 1(z-5) = 0$

9. $xyz = 10 \qquad F(x,y,z) = xyz - 10$

$\nabla F = yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k} \qquad \nabla F(1,2,5) = 10\hat{i} + 5\hat{j} + 2\hat{k}$

$10(x-1) + 5(y-2) + 2(z-5) = 0$

10. $f(x,y) = 120x + 120y - xy - x^2 - y^2$

$f_x = 120 - y - 2x = 0 \qquad 2x + y = 120$

$f_y = 120 - x - 2y = 0 \qquad 2y + x = 120 \Rightarrow$

$$\begin{array}{r} 2x + y = 2y + x \\ -x \;\; -y \;\; -y \;\; -x \\ \hline x = y \end{array}$$

$f_{xx} = -2$

$f_{yy} = -2 \qquad 2y + y = 120$

$3y = 120$

$f_{xy} = -1 \qquad y = 40$

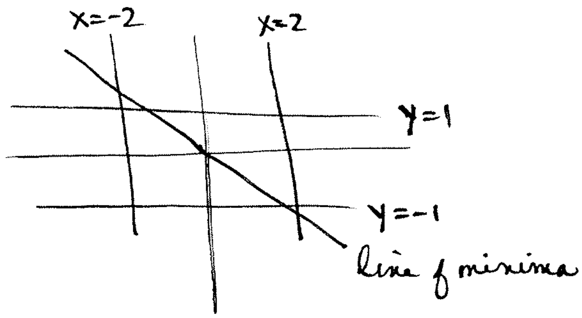
$x = 40 \qquad (40, 40)$

$D = f_{xx} f_{yy} - f_{xy}^2 = (-2)(-2) - (-1)^2 = 4 - 1 = 3 > 0$ max/min

$f_{xx} < 0$ max

this is an upside down paraboloid

11. $f(x,y) = 2xy + x^2 + y^2$ 2D min/max



$f_x = 2y + 2x = 0$

$f_y = 2x + 2y = 0 \quad \Rightarrow -x = y$

$f(x,-x) = 0$ 1D min/max

$f(2,y) = 4y + 4 + y^2 \qquad f' = 4 + 2y = 0 \Rightarrow 2y = -4 \Rightarrow y = -2$ outside region

$f(-2,y) = -4y + 4 + y^2 \qquad f' = -4 + 2y = 0 \Rightarrow 2y = 4 \Rightarrow y = 2$ outside region

$f(x,1) = 2x + x^2 + 1 \qquad f' = 2 + 2x = 0 \Rightarrow 2x = -2 \Rightarrow x = -1$ part of the line

$f(x,-1) = -2x + x^2 + 1 \qquad f' = -2 + 2x = 0 \Rightarrow 2x = 2 \Rightarrow x = 1$ of minima

$f(-1,1) = 0$

$f(1,-1) = 0$

now check corners

$f(-2,1) = 2(-2)(1) + 4 + 1 = 1$

$f(2,1) = 2(2)(1) + 4 + 1 = 9$ > absolute max

$f(-2,-1) = 2(-2)(-1) + 4 + 1 = 9$

$f(2,-1) = 2(2)(-1) + 4 + 1 = 1$

absolute minima all along the line $y = -x$ from $(-1,1)$ to $(1,-1)$ $f=0$

absolute maxima at $(2,1)$ and $(-2,-1)$ $f=9$

12. $f(x,y,z) = xyz$ s.t. $x+y+z-6=0$

$F = xyz - \lambda(x+y+z-6) = 0$

$xyz - \lambda x - \lambda y - \lambda z + 6\lambda = 0$

$F_x = yz - \lambda = 0 \Rightarrow \lambda = yz$

$F_y = xz - \lambda = 0 \Rightarrow \lambda = xz$

$F_z = xy - \lambda = 0 \Rightarrow \lambda = xy$

$F_\lambda = -x - y - z + 6 = 0$

$yz = xz \Rightarrow yz - zx = 0$ $z(y-x) = 0$

either $z=0$ or $y=x$

$yz = xy \Rightarrow yz - xy = 0 \Rightarrow y(z-x) = 0$

either $y=0$ or $x=z$

$xz = xy \Rightarrow xz - xy = 0 \Rightarrow x(z-y) = 0$

either $x=0$ or $y=z$

$y=x,\ x=z,\ y=z \Rightarrow$

$x=y=z$

$F_\lambda \Rightarrow -x-x-x+6=0$

$-3x = -6$

$x = 2 \Rightarrow (2,2,2)$

if $x=0$, $x=z \Rightarrow y=6$

if $x=0$ $y=z \Rightarrow -2y=-6$ $y=3 \Rightarrow (0,3,3)$

if $y=0$ $x=z \Rightarrow (3,0,3)$

if $z=0$ $y=x \Rightarrow (3,3,0)$

if $y=0$ $x=0$, $z=0 \Rightarrow (0,0,0)$

$f(2,2,2) = 8$ * max

$f(0,3,3) = 0$

$f(3,0,3) = 0$

$f(3,3,0) = 0$

$f(0,0,0) = 0$

The rest are minima

13. $f(x,y,z) = x^2 + y^2 + z^2$ s.t. $x + 2z = 4 \Rightarrow x + 2z - 4 = 0$

$x + y = 8 \Rightarrow x + y - 8 = 0$

$F = x^2 + y^2 + z^2 - \lambda(x + 2z - 4) - \mu(x + y - 8) = 0$

$x^2 + y^2 + z^2 - \lambda x - 2\lambda z + 4\lambda - \mu x - \mu y + 8\mu$

$F_x = 2x - \lambda - \mu = 0$ $2x = \lambda + \mu$

$F_y = 2y - \mu = 0 \Rightarrow \mu = 2y \Rightarrow 2x = 2y + z$

$F_z = 2z - 2\lambda = 0 \Rightarrow z = \lambda$

$F_\lambda = -x - 2z + 4 = 0$

$F_\mu = -x - y + 8 = 0$

$$\begin{aligned} 2x - 2y - z &= 0 \\ x + 2z &= 4 \\ x + y &= 8 \end{aligned}$$

$$\begin{aligned} 2x - \cancel{2y} - z &= 0 \\ 2x + \cancel{2y} &= 16 \\ \hline 2x - z &= 16 \end{aligned}$$

$$\begin{aligned} 4x - 2z &= 32 \\ x + 2z &= 4 \\ \hline \frac{5x}{\cancel{5}} &= \frac{36}{5} \end{aligned}$$

$x = \frac{36}{5}$

$y = 8 - \frac{36}{5} = \frac{64}{5} - \frac{36}{5} = \frac{28}{5}$

$z = 2\left(\frac{36}{5}\right) - 16 = \frac{72}{5} - 16 = \frac{72}{5} - \frac{80}{5} =$

$-\frac{8}{5}$

$\left(\frac{36}{5}, \frac{28}{5}, -\frac{8}{5}\right)$

14. a) $\int_{\pi/2}^{\pi} \int_1^2 x\cos(xy)\,dy\,dx = \int_{\pi/2}^{\pi} \frac{\cancel{x}\sin(xy)}{\cancel{x}}\Big|_1^2\,dx =$

$\int_{\pi/2}^{\pi} \sin(2x) - \sin(x)\,dx = -\frac{1}{2}\cos(2x) + \cos(x)\Big|_{\pi/2}^{\pi} =$

$-\frac{1}{2}(1) + (-1) + \frac{1}{2}(-1) - 0 = -\frac{1}{2} - 1 - \frac{1}{2} = -2$

b. $\iint\limits_R \frac{xy}{\sqrt{x^2+y^2+1}}\,dA = \int_0^1\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}}\,dx\,dy$

$u = x^2+y^2+1$

$\frac{1}{2}du = \cancel{2}x\,dx$

$\int \frac{xy}{\sqrt{x^2+y^2+1}}\,dx = \int \frac{1}{2}\frac{y\,du}{\sqrt{u}} =$

$\frac{1}{2}y\int u^{-1/2}\,du = y u^{1/2} = y(x^2+y^2+1)^{1/2}\Big|_0^{x=1} = y(2+y^2)^{1/2} - y(1+y^2)^{1/2}$

$\int_0^1 y(2+y^2)^{1/2} - y(1+y^2)^{1/2}\,dy$ $\quad u = 2+y^2 \quad w = 1+y^2$

$\frac{1}{2}du = \cancel{2}y\,dy \quad \frac{1}{2}dw = \cancel{2}y\,dy$

$\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}(2+y^2)^{3/2} - \frac{1}{\cancel{2}}\frac{\cancel{2}}{3}(1+y^2)^{3/2}\Big|_0^1 =$

$\frac{1}{3}(2+1)^{3/2} - \frac{1}{3}(1+1)^{3/2} - \frac{1}{3}(2)^{3/2} + \frac{1}{3}(1)^{3/2} =$

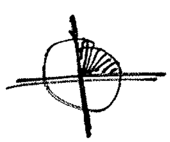
$\boxed{\frac{1}{3}(3)^{3/2} - \frac{2}{3}(2)^{3/2} + \frac{1}{3}}$

15. $\int_1^3\int_0^2 3x^3 + 3x^2y\,dy\,dx = \int_1^3 3x^3y + \frac{3}{2}x^2y^2\Big|_0^2\,dx =$

$\int_1^3 6x^3 + 6x^2\,dx = \frac{6}{4}x^4 + 2x^3\Big|_1^3 = \frac{3}{2}(81) + 2(27) - \frac{3}{2}(1)^4 - 2(1)^4 =$

$\frac{243}{2} + 54 - \frac{3}{2} - 2 = \frac{240}{2} + 52 = 120 + 52 = \boxed{172}$

16. $f(x,y) = e^{-(x^2+y^2)}$ over $R: x^2+y^2 \le 4 \quad x \ge 0 \quad y \ge 0$



Convert to polar

$f(r,\theta) = e^{-r^2} \quad 0 \le r \le 2 \quad 0 \le \theta \le \pi/2$

$\int_0^{\pi/2} \int_0^2 e^{-r^2} r \, dr \, d\theta = \int_0^{\pi/2} -\frac{1}{2} e^{-r^2} \Big|_0^2 \, d\theta = \int_0^{\pi/2} \left[-\frac{1}{2} e^{-4} + \frac{1}{2}(1) \right] d\theta$

$= \frac{1}{2}\left(1 - \frac{1}{e^4}\right) \int_0^{\pi/2} d\theta = \frac{1}{2}\left(1 - \frac{1}{e^4}\right) \frac{\pi}{2} = \frac{\pi}{4}\left(1 - \frac{1}{e^4}\right)$

17. $z = x^2 + y^2 + 1$, $z = 0$, $x^2 + y^2 = 4$ again, use polar

$z = r^2 + 1$, $z = 0$, $r = 2$

$\int_0^{2\pi} \int_0^2 (r^2 + 1) r \, dr \, d\theta = \int_0^{2\pi} \int_0^2 r^3 + r \, dr \, \theta = \int_0^{2\pi} \frac{1}{4} r^4 + \frac{1}{2} r^2 \Big|_0^2 d\theta$

$= \int_0^{2\pi} \frac{1}{4}(16) + \frac{1}{2}(4) \, d\theta = \int_0^{2\pi} 4 + 2 \, d\theta = \int_0^{2\pi} 6 \, d\theta = 6 \cdot 2\pi = \boxed{12\pi}$

18. a) $f(x,y) = 10 + 2x + 2y$

$f_x = 2$

$f_y = 2$

$SA = \int_0^2 \int_0^2 \sqrt{1 + 4 + 4} \, dx \, dy = \int_0^2 \int_0^2 \sqrt{9} \, dx \, dy =$

$\int_0^2 \int_0^2 3 \, dx \, dy = \boxed{12}$

b) $f(x,y) = 2 + \frac{2}{3} x^{3/2}$

$f_x = x^{1/2}$

$f_y = 0$

$SA = \int_0^1 \int_0^{1-x} \sqrt{1 + (x^{1/2})^2 + 0^2} \, dy \, dx =$

$= \int_0^1 \int_0^{1-x} \sqrt{1 + x} \, dy \, dx = \int_0^1 y \, (1+x)^{1/2} \Big|_0^{1-x} dx$

$\int_0^1 (1-x)\sqrt{1+x} \, dx$

$u = 1-x \qquad dv = \sqrt{1+x}$

$du = -\,dx \qquad v = \frac{2}{3}(1+x)^{3/2}$

$$(1-x)\left(\frac{2}{3}\right)(1+x)^{3/2} + \int_0^1 \frac{2}{3}(1+x)^{3/2}\,dx = (1-x)\left(\frac{2}{3}\right)(1+x)^{3/2} + \frac{2}{3}\cdot\frac{2}{5}(1+x)^{5/2}\Big|_0^1 =$$

$$0 + \frac{4}{15}(2)^{5/2} - \frac{2}{3}(1)^{3/2} - \frac{4}{15}(1)^{5/2} = \frac{4}{15}4\sqrt{2} - \frac{5\cdot 2}{5\cdot 3} - \frac{4}{15} = \boxed{\frac{16}{15}\sqrt{2} - \frac{14}{15}}$$

$-10/15 \quad -4/15$

c) $f(x,y) = xy$

$f_x = y$

$f_y = x$

$$SA = \iint_{R = x^2+y^2 \le 16} \sqrt{1+y^2+x^2}\,dA \qquad \text{switch to polar}$$

$$= \int_0^{2\pi}\int_0^4 \sqrt{1+r^2}\; r\,dr\,d\theta \qquad u = 1+r^2 \quad \frac{1}{2}du = \cancel{2}r\,dr$$

$\frac{1}{2}u^{1/2}\,du$

$$\frac{1}{\cancel{2}}\int_0^{2\pi} \frac{\cancel{2}}{3}(1+r^2)^{3/2}\Big|_0^4\,d\theta =$$

$$\frac{1}{3}\int_0^{2\pi} (17)^{3/2} - 1^{3/2}\,d\theta = \boxed{\frac{17^{3/2}-1}{3}\cdot 2\pi}$$

d) $z = \sqrt{x^2+y^2}$ inside $x^2+y^2=1$ convert to polar

$z = r$

$r \le 1$

$$z_x = \frac{x}{\sqrt{x^2+y^2}} = \frac{\cancel{r}\cos\theta}{\cancel{r}} = \cos\theta$$

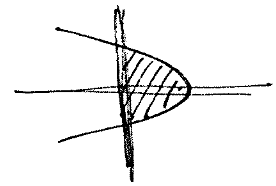
$$z_y = \frac{y}{\sqrt{x^2+y^2}} = \frac{\cancel{r}\sin\theta}{\cancel{r}} = \sin\theta$$

$$SA = \int_0^{2\pi}\int_0^1 \sqrt{1+\underbrace{\cos^2\theta+\sin^2\theta}_{=1}}\; r\,dr\,d\theta = \int_0^{2\pi}\int_0^1 \sqrt{2}\,r\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{\sqrt{2}}{2}r^2\Big|_0^1\,d\theta = \int_0^{2\pi}\frac{\sqrt{2}}{2}\,d\theta = \frac{\sqrt{2}}{\cancel{2}}\cdot\cancel{2}\pi = \boxed{\sqrt{2}\,\pi}$$

19.

$$M = \int_{-3}^{3}\int_{0}^{9-y^2} kx \, dx\, dy = \int_{-3}^{3} \frac{k}{2}x^2 \Big|_0^{9-y^2} dy =$$



$$\frac{k}{2}\int_{-3}^{3} 81 - 18y^2 + y^4 \, dy = k\int_0^3 81 - 18y^2 + y^4 \, dy =$$

even function $\Rightarrow$

$$k\left[81y - 6y^3 + \frac{1}{5}y^5\right]_0^3 = k\left[243 - 162 + \frac{243}{5}\right] = \frac{648}{5}k$$

$$M_x = \int_{-3}^{3}\int_{0}^{9-y^2} kxy \, dx\, dy = \int_{-3}^{3} \frac{k}{2}x^2 y \Big|_0^{9-y^2} dy = \frac{k}{2}\int_{-3}^{3} y(81 - 81y^2 + y^4)\, dy$$

$$= \frac{k}{2}\int_{-3}^{3} 81y - 81y^3 + y^5 \, dy \quad = 0$$

odd function $\Rightarrow$

$$M_y = \int_{-3}^{3}\int_{0}^{9-y^2} kx^2 \, dx\, dy = \int_{-3}^{3} \frac{k}{3}x^3 \Big|_0^{9-y^2} dy = \frac{k}{3}\int_{-3}^{3} 729 - 243y^2 + 27y^4 - y^6 \, dy$$

even function

$$\frac{2k}{3}\int_0^3 729 - 243y^2 + 27y^4 - y^6 \, dy = \frac{2k}{3}\left[729y - 81y^3 + \frac{27}{5}y^5 - \frac{1}{7}y^7\right]_0^3 =$$

$$\frac{2k}{3}\left[\cancel{2187} - \cancel{2187} + \frac{6561}{5} - \frac{2187}{7}\right] = \frac{2k}{3}\left[\frac{34992}{35}\right] = \frac{23328}{35}k$$

$$\bar{x} = \frac{M_y}{M} = \frac{23328k}{\cancel{35}\,7} \cdot \frac{\cancel{5}}{648k} = \frac{36}{7}$$

$$\bar{y} = \frac{M_x}{M} = \frac{0}{(648/5\,k)} = 0$$ This makes sense because the region is symmetric around the x-axis.

$$(\bar{x}, \bar{y}) = \left(\frac{36}{7}, 0\right)$$