Exploring the Energy Balance Model for Planetary Variability

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Abstract

Climate models were designed to help us understand climate change here on Earth, but they can also help us understand climates on other planets, both in our Solar System (like Venus and Mars), as well as outside it with the ever-increasing number of exoplanet discoveries. This paper examines the impacts of common planetary properties on the resulting climates of terrestrial planets, and then uses that to produce a climate model of Venus and Mars. In addition, parameter settings considerably different from that of the Earth are combined to produce a theoretical climate for a hypothetical habitable exoplanet.

1 Introduction

This project employs the energy balance model (EBM) introduced in class. [NMS83] The goal of this project is two-fold. First, to reproduce planetary environments for known terrestrial objects such as Venus and Mars, and second, to find a combination of settings that are non-earth-like, but nonetheless produce a habitable environment similar to those on present-day Earth. In order to achieve the latter goal, we will first explore several parameters in the model to determine their impact on the energy balance model. Of particular interest are parameters that contribute to heating or cooling of the global mean temperature, and any properties that have little or no impact on it.

2 Methods

2.1 Model Description

The basic equation for the energy balance in ebm.m at each location is

$$c\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2} + D\frac{\partial^2 T}{\partial y^2} + (1-\alpha)S - (a+bT)$$

The left side of the equation and the first two terms on the right side represent diffusive heat transfer that is used to simplify much of the complexity of a more complete atmospheric model. The third term on the right represents the incoming radiation, and the final term the outgoing radiation. The diffusive heat transfer does not take height into account in the model, only longitude and latitude.

Incoming radiation (insolation) depends on the solar constant, eccentricity and obliquity. Albedo depends on latitude but does not depend on temperature. Outgoing radiation is modeled as a linear function that depends on temperature. This is fine far away from a tipping point, but as we will see when parameters are altered, this is approximately valid only for solutions near equilibrium.

2.2 Parameter Testing

The first step in achieving the goal of modeling a habitable exoplanet is to test the planetary parameters of the model to determine the impact on global mean temperatures. Parameters that will be tested include:

1. The planetary mask
2. Incoming solar radiation and variability
3. Outgoing Longwave Radiation
4. Obliquity (Axial Tilt)
5. Eccentricity (orbital)
6. Planet size
7. Length of the Year

Several parameter settings will be tested for each of these variables to determine their impact on the model. All parameters except the one being tested will remain fixed. Parameter combinations for Venus and Mars will be tested to see how the model performs for known planets. Finally, we will attempt to find one or more combinations of parameters that can provide a mean global temperature suitable for humans, or as close to it as possible with parameters distinctly different from those on the real Earth.

3 Results

3.1 Effect of Planetary Mask

The first thing to find out is how strong an effect the details of planetary surfaces might have on a global energy balance model. Starting from the Earth mask which is roughly 75% water and 25% land, I reversed the values to produce a mask that is 75% land and 25% water. The specific configuration of the continents did not concern me here. The two other masks used were generated from random numbers (initially between 0 and 1, and then rescaled). One cell was given a value at the opposite end of the scale so that the scales for all the masks would remain the same. The maps of the four masks are shown in Figure 1.

The energy balance model was run for the equivalent of twenty years, and the global mean temperature was calculated for the final year of the run. This data is shown in Table 1. In Figure 2, the global mean temperatures for the entire run is shown. As you can see from the graph and the table, the masks have very little effect on the global mean temperature after this length of time. The Ocean World mask does require more time to reach equilibrium, indeed, it probably could be run a bit longer to obtain a more accurate final value. Even so, the differences between each of the masks is less than 0.03 degrees Celsius. This suggests that, at least for this model, the specific surface configuration does not have a large effect on the model in the long-term, and so any of the masks can be used more-or-less interchangeably for the purposes of this project.

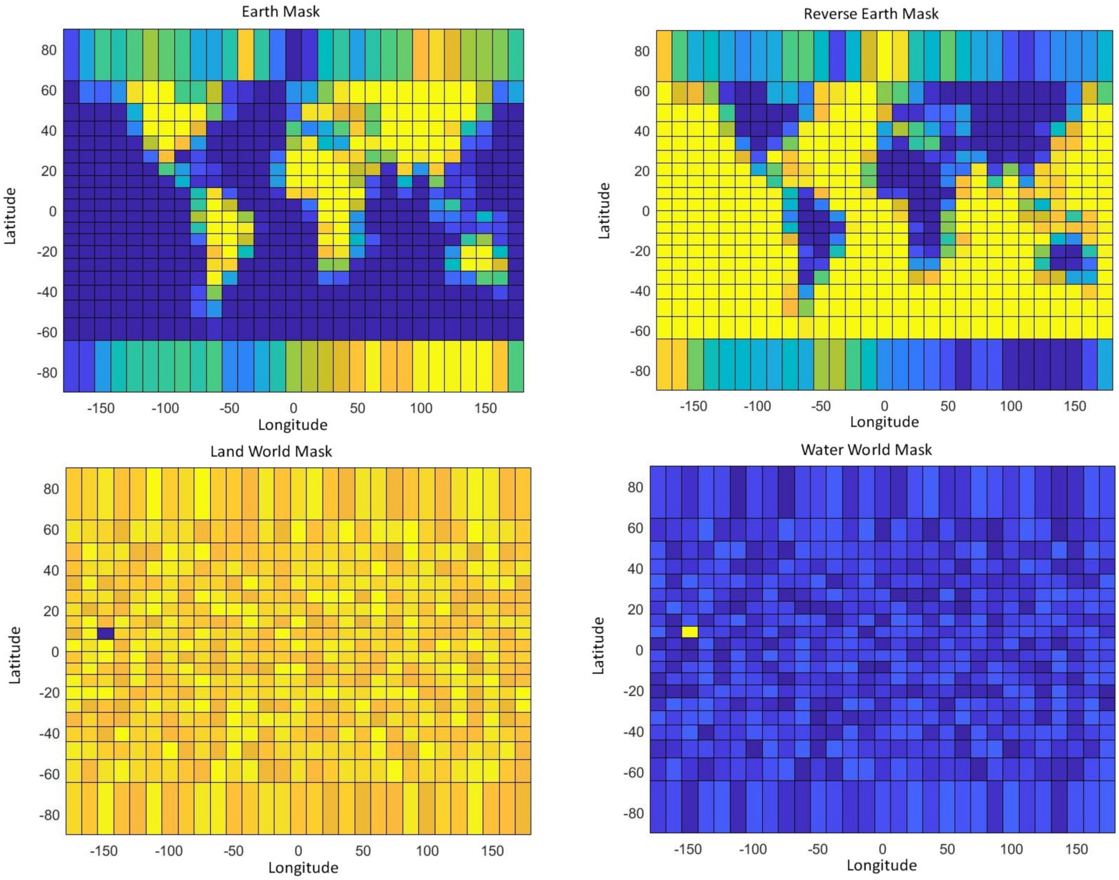


Figure 1: Maps of four masks tested in model: 1. Earth, 2. Reverse Earth, 3. Land Mask, 4. Ocean Mask.

Table 1: Global Mean Temperatures After 20 Years for Different Planetary Masks

Mask	Earth	Reverse Earth	Ocean	Land
Global Mean Temperature	18.1652	18.1674	18.1892	18.1652

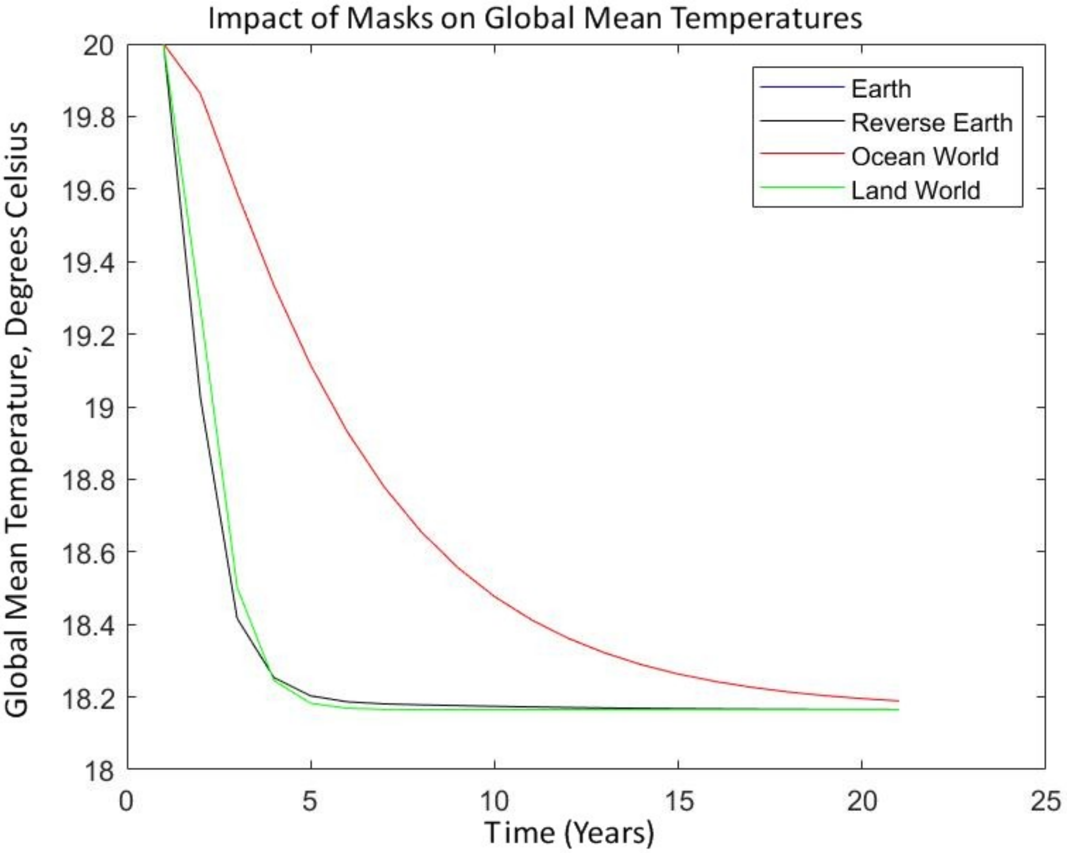


Figure 2: Impacts on Global Mean Temperatures of using each of the four masks.

Table 2: Effect on Global Mean Temperatures as solar output changes

Epoch	Solar Constant (Wm^-2)	Global Mean Temperature (Celsius)
4 billion years ago	928.9	-18.2078
3 billion years ago	1021.8	-10.3629
2 billion years ago	1124.0	-1.7419
1 billion years ago	1236.4	7.7395
Now	1360.0	18.1656
1 billion years from now	1496.0	29.6377
2 billion years from now	1645.6	42.2570
3 billion years from now	1810.2	56.1417
4 billion years from now	1991.2	71.4097
5 billion years from now	2190.3	88.2045

3.2 Effect of Solar Variability

There are a number of components to solar variability that might be useful to incorporate into a planetary or exo-planetary model. Our Sun undergoes an eleven-year solar cycle. However, the amplitude of that variability for the Sun is only about 2% of the total output. And to see its long-term effect, we would have to run the model for a much longer period of time. Some stars have much more extreme variability, particularly red dwarfs around which many Earth-size planetary objects have been found. Habitable Zones are much closer to the stars than Earth is because the stars are dimmer and smaller, but this also makes them more subject to stellar volatility. [Pie10]

Another important component of solar variability is that the energy output of the Sun changes over time. We know that the Earth has had life for billions of years, and this output changes about 10% per billion years. [Pie10] Over the life of the Earth, this change is significant. Table 2 shows the estimated incoming solar energy at the Earth's orbit backwards and forwards in time, and the global mean temperature predicted by the energy balance model. Figure 3 and Figure 4 show the line graph of global mean temperatures over the model run, and the final global mean temperature for the final year displayed as a bar graph, respectively.

While we can be skeptical of the specific temperatures predicted here, it is easy to see how the equilibrium temperatures are impacted by changes in solar output. The atmosphere may retain more heat by including more greenhouse gases, for instance, it's not hard to see how speculation about Snowball Earth could be justified in the past. It is equally easy to see how predictions of the far-future Earth could lead to seas being boiled away into space.

Given that we have the three values from the solar constant for Venus, Earth and Mars, we can use curve fitting or regression to obtain a model for the solar constant given a distance from the Sun. If we fit a power model with constant, we obtain the model $y = 2.296 \times 10^5 x^{-0.9031} - 2472$ (x is distance in millions of miles), or if we just apply a power model regression, we get $y = 1.465 \times 10^8 x^{-2.59} (R^2 = 0.969)$. From this information we could estimate the expected solar radiation an imaginary planet receives at a given distance from the Sun. Or it's inverse: given a solar constant value, estimate the equivalent distance from our Sun. When comparing climates around other stars, it can be useful to compare where that would be relative to a system we know much better.

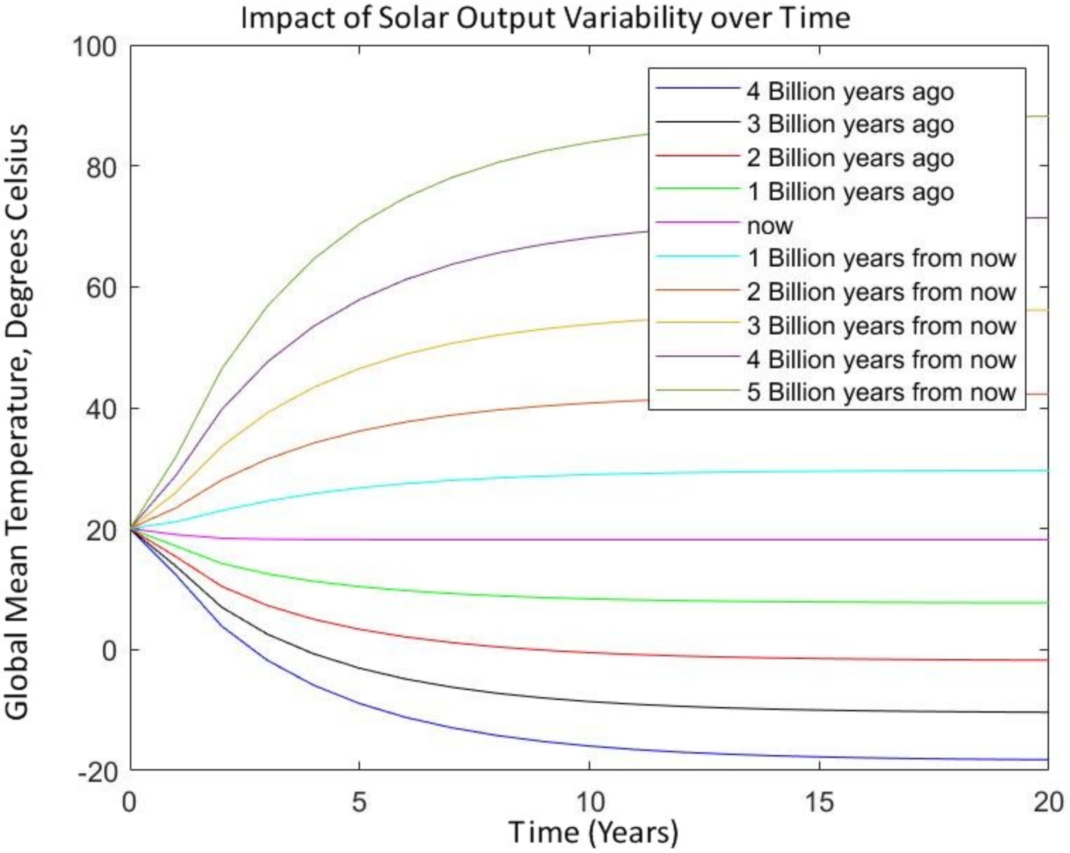


Figure 3: Impacts on Global Mean Temperatures of Solar Output Changes over Lifetime of the Sun

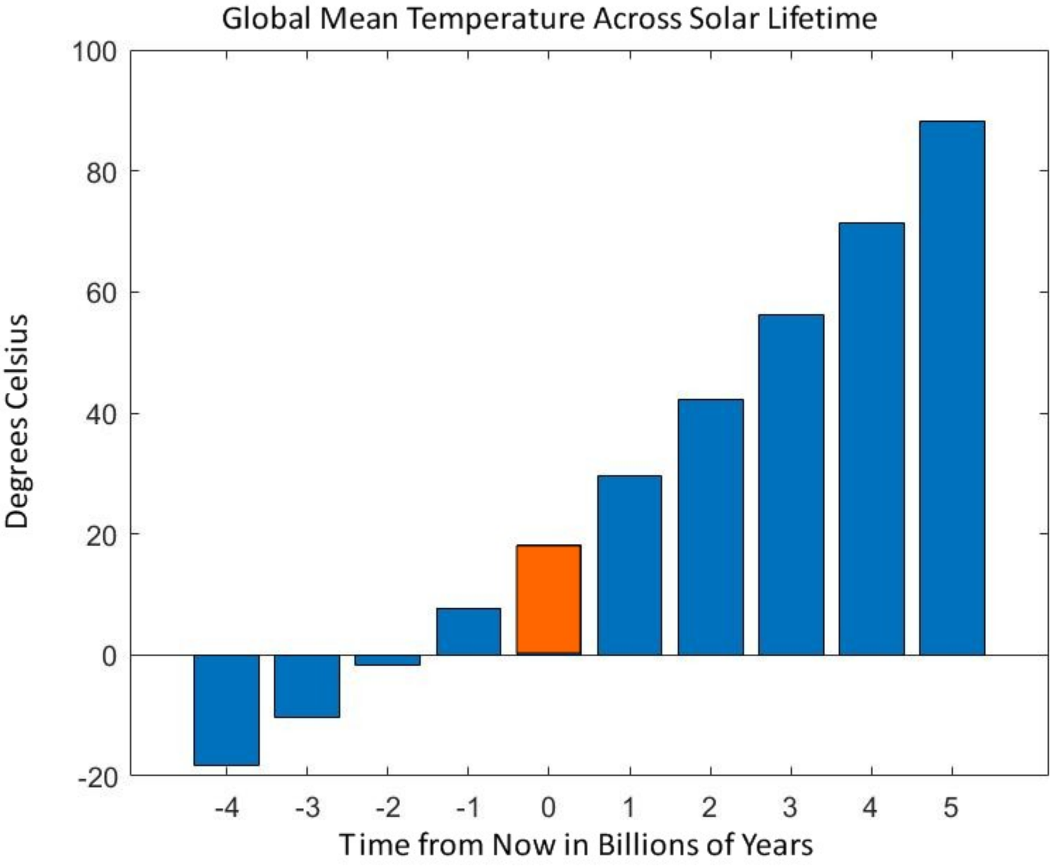


Figure 4: Global Mean Temperatures Across Solar Lifetime at Equilibrium

Table 3: Effect on Global Mean Temperatures as coefficients of outgoing radiation changes

Constant a (Wm^-2)	Global Mean Temp	Coefficient b (Wm^-2K^-1)	Global Mean Temp (Celsius)
2000	-835.9636	20	1.9795
203.3	18.1656	2.094	18.1656
20	105.3042	0.2	91.0123
		0.02	121.8792

3.3 Effect of Outgoing Radiation

To establish some general behavior for the effect of changing the outgoing radiation parameters, we will start by comparing the effect of modifying to two parameters, one at a time, but an order of magnitude in each direction, and comparing the results with the baseline values for earth.

Let us start with the constant a in the equation. The initial value for the earth is given in the model as 203.3 Wm^-2, and we will increase that to 2000 Wm^-2, and reduce it to 20 Wm^-2. The results are shown in the first two columns of Table 3. Then we will likewise adjust the b value in the equation that expresses how the outgoing radiation depends on temperature. Those results are displayed in the last two columns of Table 3. This data is also displayed as graphs in Figure 5 and Figure 6. respectively.

As one can see from the Figure 5, increasing the constant decreases the temperature. The largest value tested here, 2000 Wm^-2, is not realistic since this more than the incoming radiation, but qualitatively, this does not change the direction of the effect. From Figure 6, we can see that large coefficients not only impact the speed at which equilibrium is achieved (smaller values take much longer than 20 cycles of the model), but the global mean temperatures also increase as the coefficient decreases (an inverse relationship).

3.4 Effect of Obliquity

One of the planetary parameters that we humans get to experience all the time is the axial tilt of the Earth, or its obliquity. This is the property of the Earth that produces the seasons, and not all planets have similar obliquity. Venus, though it rotates retrograde, has virtually no axial tilt, while Mars has a tilt that is quite similar to the Earth's, and Uranus has essentially a 90-degree tilt, meaning that one pole or the other could be pointing directly away from the Sun, putting one hemisphere into total darkness.

One way of visualizing this is to look at a graph of insolation, which is the amount of sun being received by the surface at a given latitude at a given time of the year. As shown in Figure 7, when there is no axial tilt (obliquity), the amount of solar energy received by the surface remains constant all year. However, as the obliquity increases, there begin to be parts of the surface away from the poles that receive less sunlight during part of the year, and more sunlight at other parts of the year. As the obliquity increases, the regions of high sun shift away from the equator, until at 90-degree tilt, the poles receive both long periods of darkness and long periods of direct sunlight.

Unlike for changing the total amount of incoming solar radiation, it is less clear what the impact of the obliquity is likely to be on global mean temperatures. Figure 8 shows the results of various obliquity values at roughly regular intervals. What is striking here is that the 90-degree tilt appears to have less impact on global mean temperatures than does the 0-degree tilt. It's possible that this effect is not entirely real, and is the result of the way that albedo is calculated by latitude in this model.

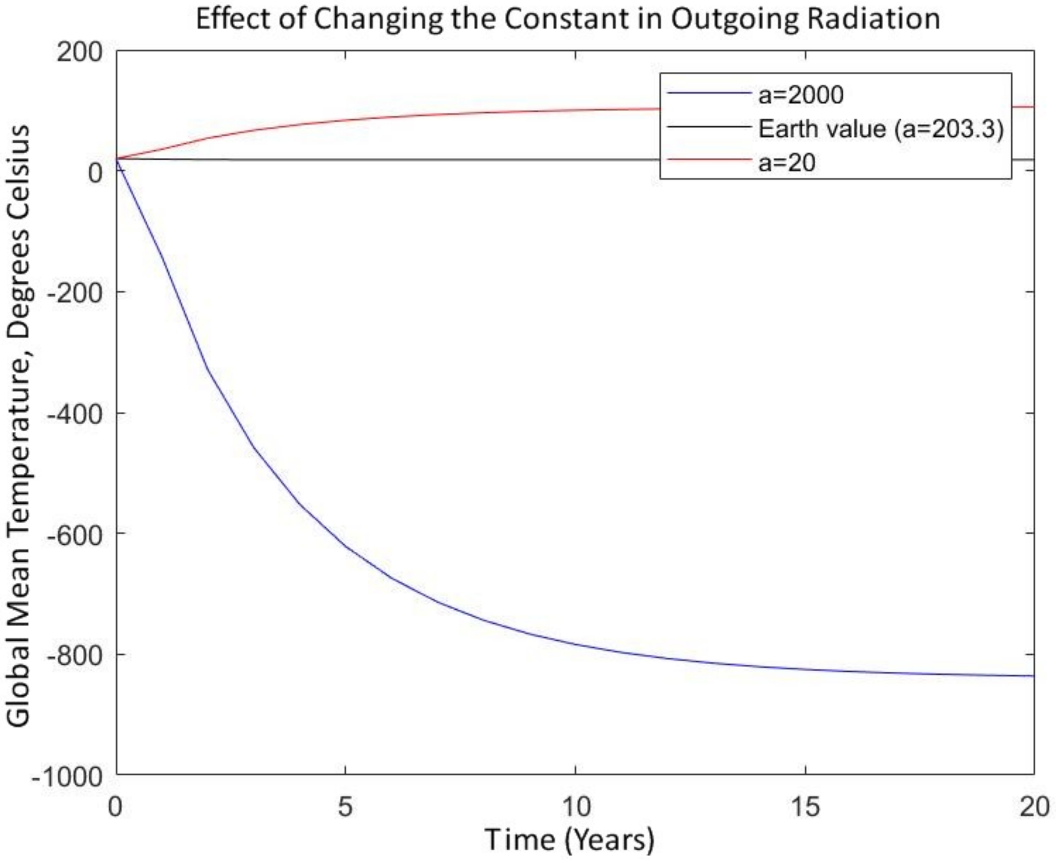


Figure 5: Global Mean Temperatures effects of changing outgoing radiation constant

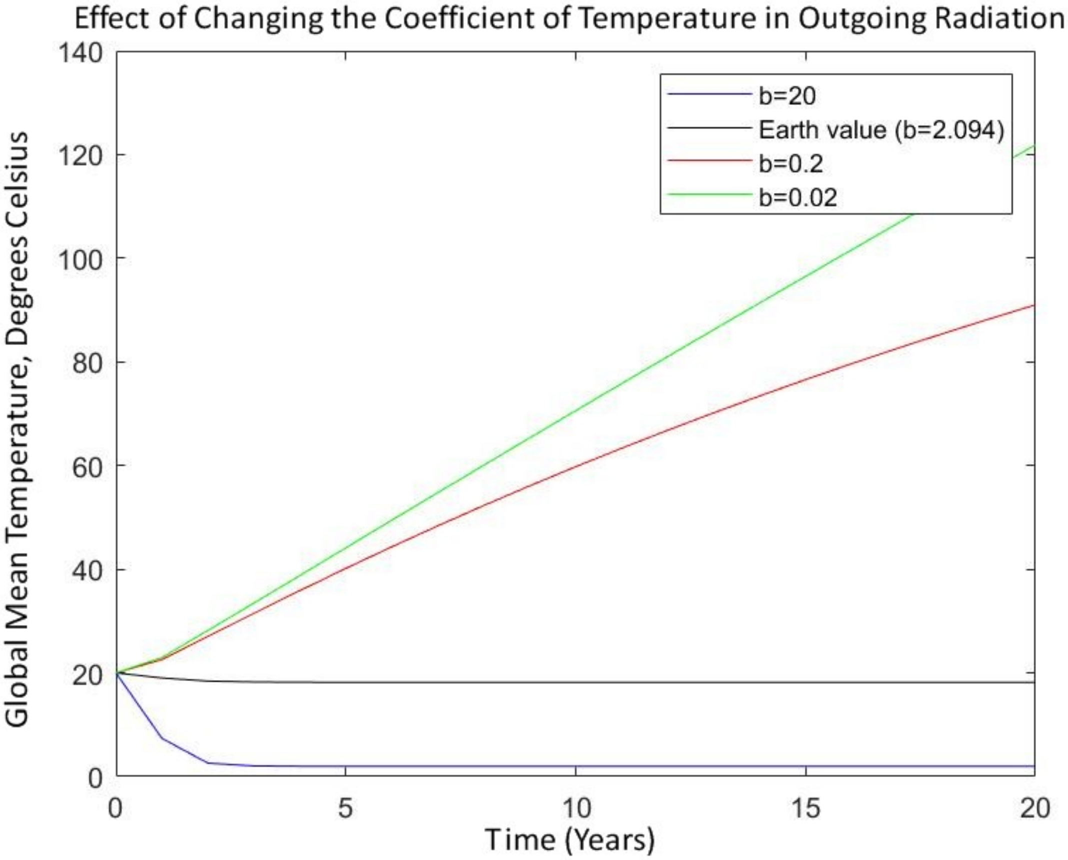
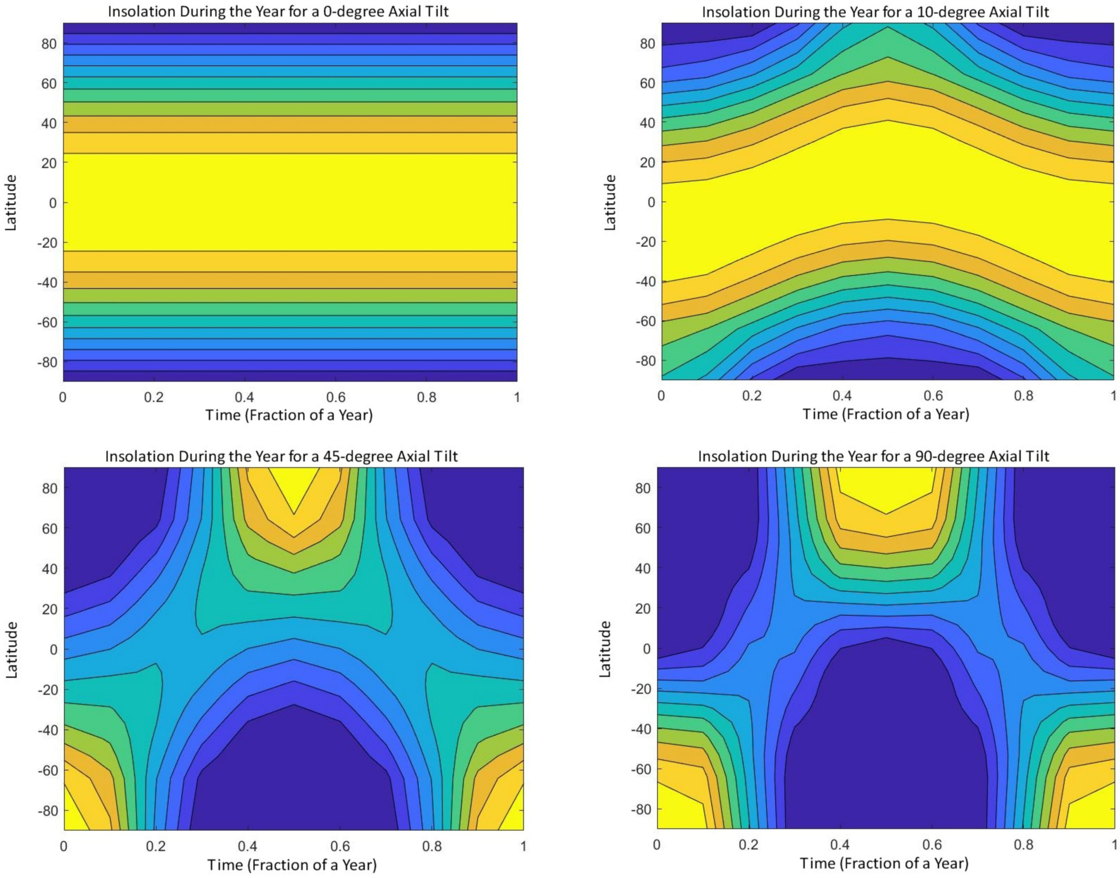


Figure 6: Global Mean Temperatures effects of changing outgoing radiation coefficient of temperature



Insolation During the Year for a 0-degree Axial Tilt

Insolation During the Year for a 10-degree Axial Tilt

Insolation During the Year for a 45-degree Axial Tilt

Insolation During the Year for a 90-degree Axial Tilt

Figure 7: Insolation by Latitude and Time of Year for four Axial Tilts

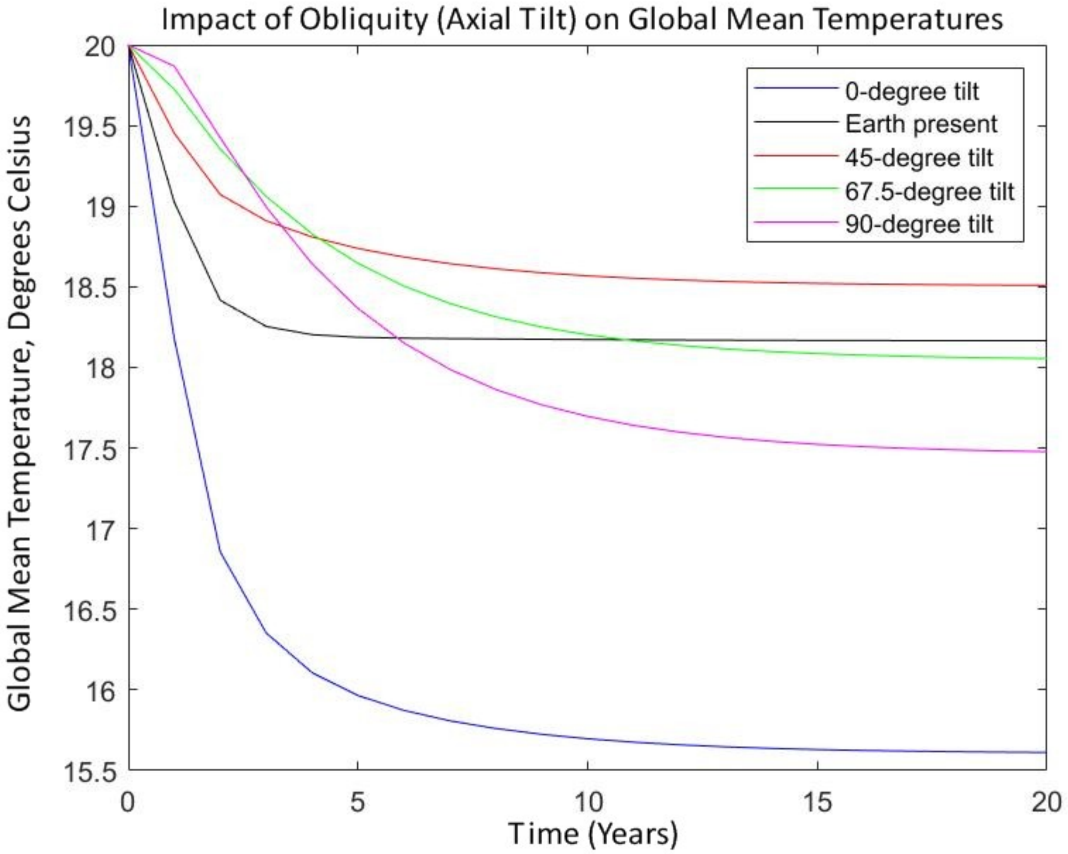


Figure 8: Impact of Obliquity on Global Mean Temperatures

Table 4: Effect on Global Mean Temperatures as solar output changes

Planet(s)	Eccentricity	Global Mean Temperature (Celsius)
Perfect Circle	0	14.3623
Earth	0.017	18.1656
Gas Giants (Uranus, Jupiter, and Saturn)	≈ 0.05	25.7314
Mars	0.0934	36.0494
Mercury	0.2056	64.6604
Eris	0.44	133.4438
Sedna	0.855	285.1156

3.5 Effect of Eccentricity

Eccentricity is a property of a planet's orbit. Zero eccentricity is a perfect circle, while larger values (up to but not including one) represent more elongated, elliptical orbits. Some sample eccentricity values are listed in Table 4. The Eccentricity of the Earth's orbit is quite small, so that its orbit is nearly circular. Other planets, such as Mars, are more elliptical. Mercury and Pluto have somewhat similar eccentricities, while Kuiper Belt Objects have the largest eccentricities in the Solar System. As the orbits become more elliptical, the changing distance from the Sun at different parts of the orbit could have an impact on global mean temperatures. It was not immediately clear before running the model whether the close approach to the Sun would have a larger or small effect than the longer periods of the year spent further away from the Sun. The results of running the model for various eccentricity values is shown in Figure 9.

From Figure 9, it is possible to see that the variability in temperatures also increases as eccentricity increases.

3.6 Effect of Size of Planet and Length of Year

The size of the planet has no effect on this energy balance model because units used to account for both incoming and outgoing radiation already account for unit area. Changing the size of the planet had no direct impact on this model, at least not directly. Of course, larger planets may retain thicker atmospheres with different composition, but that would have to be accounted for in other ways.

Likewise, the length of the year had no impact on the equilibrium global mean temperature. Of course, adjusting the length of the year in the real world would entail moving closer to the central star and thus produce more radiation, but if we can imagine a world around a dimmer star with a shorter year receiving the same amount of radiation, it's the energy from the star that matters, not the length of the year directly.

3.7 Reconstructing the Climates of Venus and Mars

The next step is to model Venus and Mars with the Energy Balance model. Several sources ([Tit+07] [05] [Mad+12] [Kni15]) provided parameter values, and then one final parameter in the model was tweaked to obtain the correct mean surface temperatures. Table 5 shows the parameter settings for each of the models, and the plot of global mean temperatures for the final model run for each planet is shown in Figure 11.

Parameters not listed here were not adjusted, and this include the parameters around albedo. This is perhaps less of a problem for Mars. However, the clouds of Venus are quite bright, and so very little energy

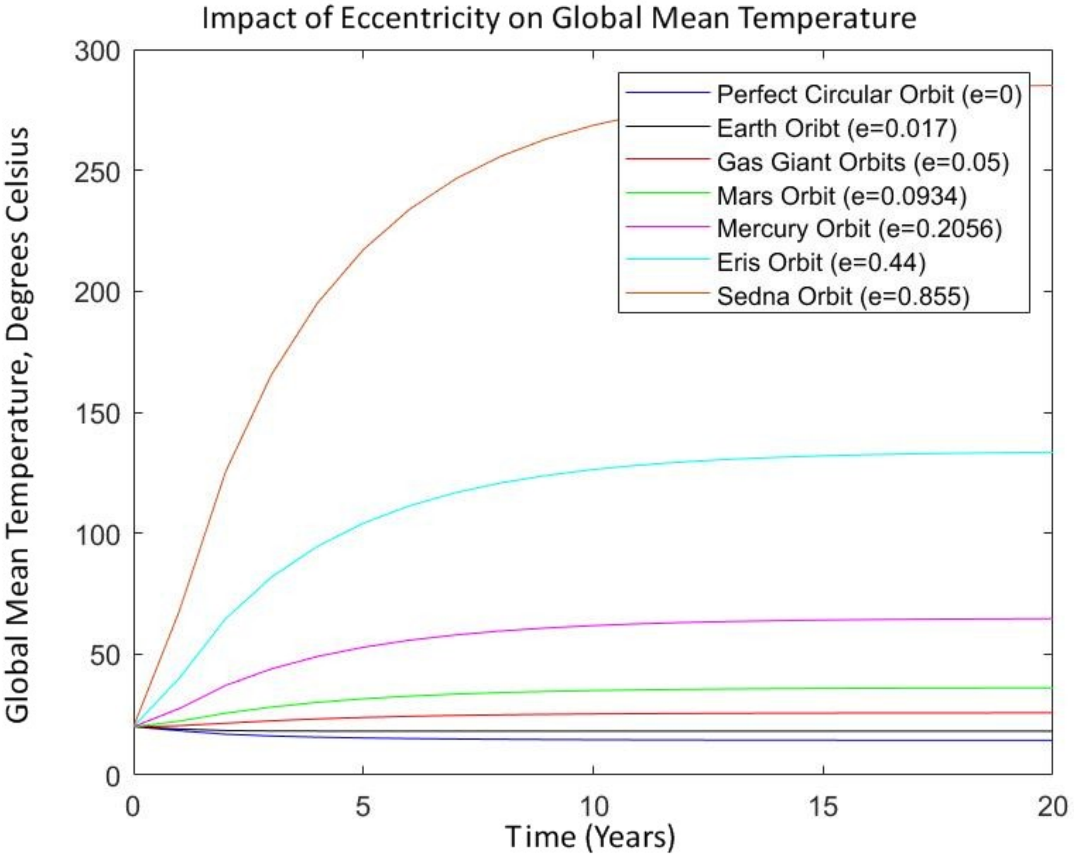


Figure 9: Orbital Eccentricity Impacts on Global Mean Temperature

Table 5: Parameter Settings for Energy Balance Models for Earth, Venus and Mars

Parameters	Earth	Venus	Mars
Mask	Earth	Land	Land
Initial Temperature	20.0	450.0	-63.0
Obliquity	23.45	177.36	25.19
Year	31556926.0	21142166.4	59354294.4
Eccentricity	0.17	0.006772	0.0934
Solar Constant	1360.0	2622	148.5
Longwave a	203.3	15.3	32.2
Longwave b	2.094	0.6	0.03
Measured Global Mean Temperature	14	464	-63
Predicted Global Mean Temperature	18.1656	464.1996	-63.1068

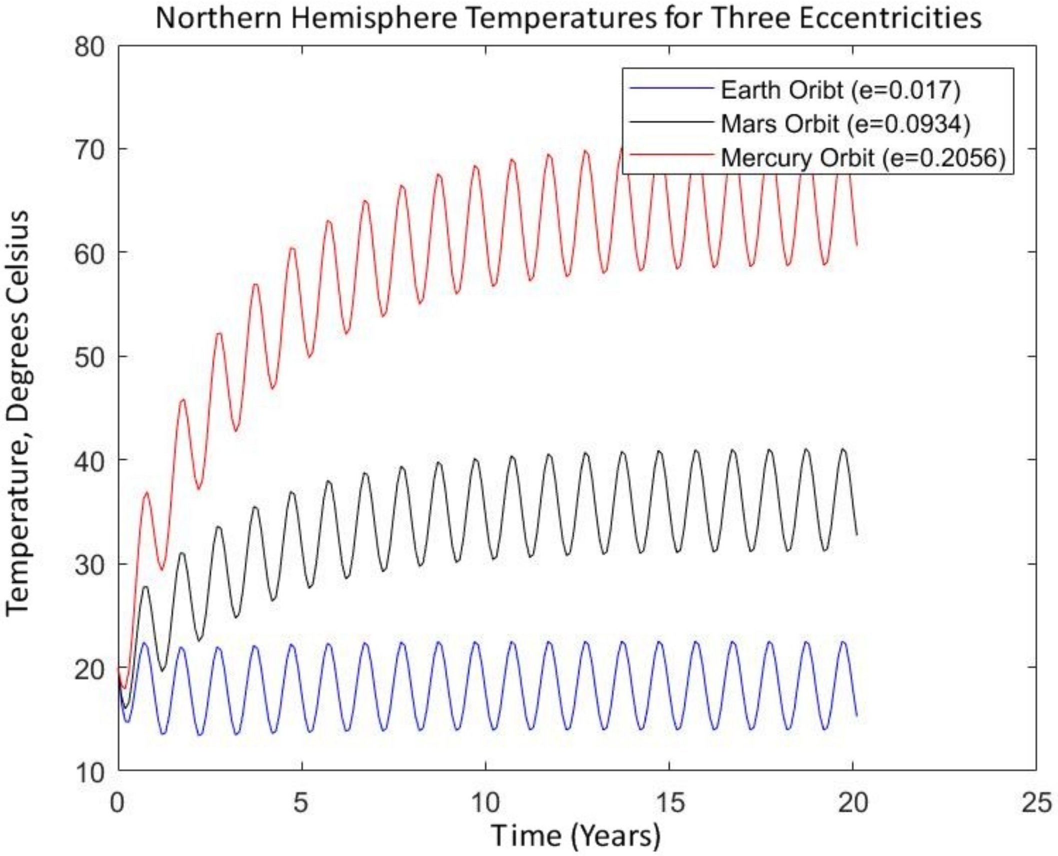


Figure 10: Effect of Orbital Eccentricity on Northern Hemisphere Temperature Variation

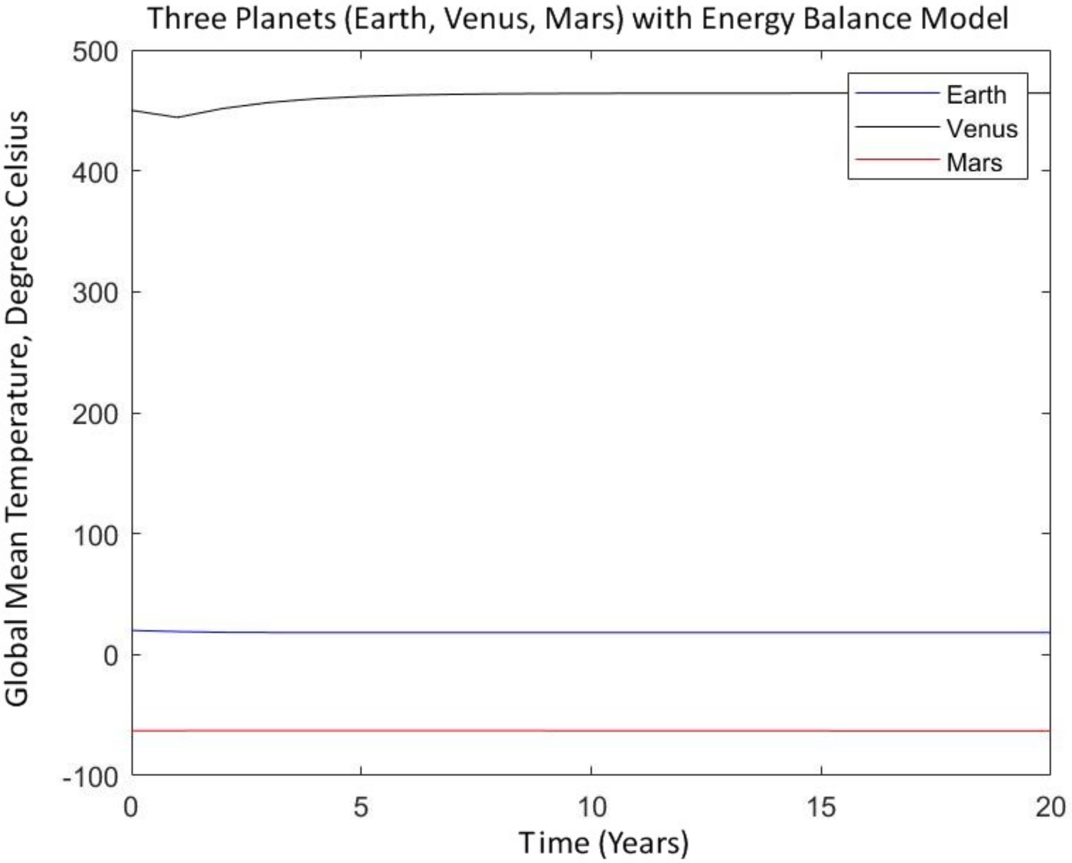


Figure 11: Effect of Orbital Eccentricity on Northern Hemisphere Temperature Variation

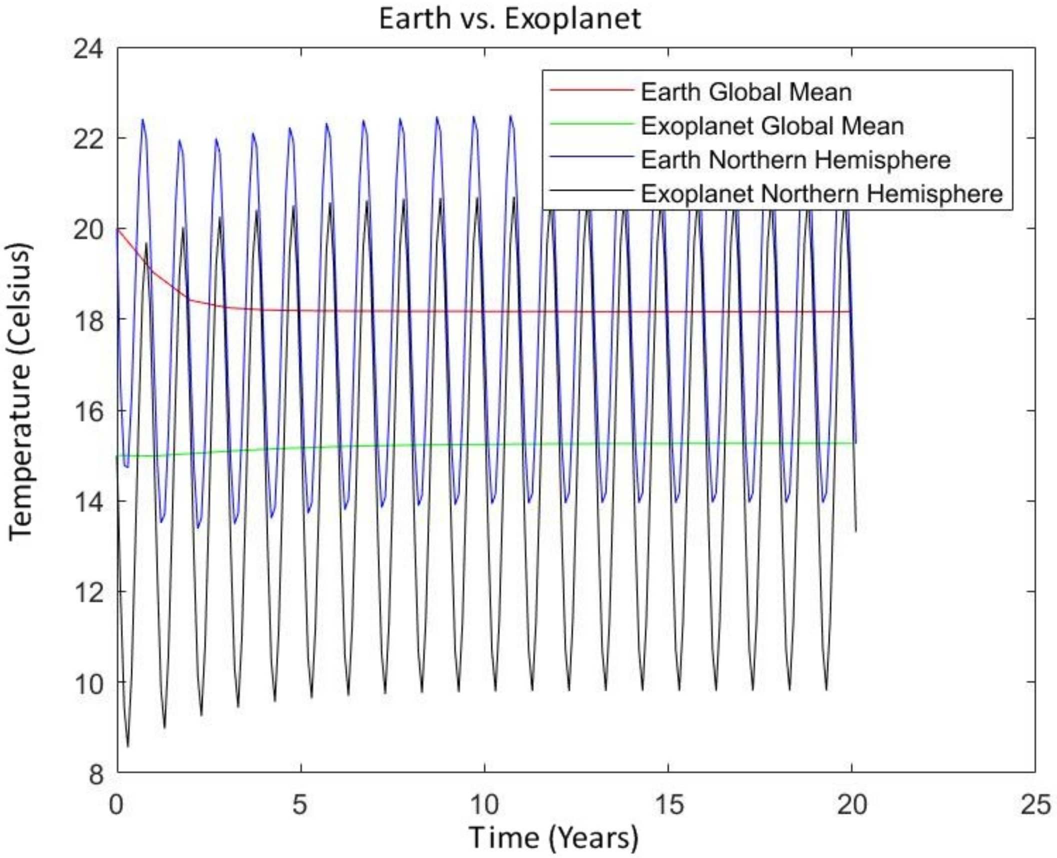


Figure 12: Comparing the Real Earth vs. a Habitable Exoplanet Model

reaches the surface to be absorbed, but the density carbon dioxide atmosphere absorbs enough to maintain the temperature as high as it is.

3.8 Building a Habitable Exoplanet Climate

To build a habitable exoplanet climate, I wanted to begin with parameter settings substantially different from that of the Earth, to establish the variety of combinations of settings that could produce a habitable climate. The parameter settings chosen for the example model is shown in Table 6. I used a water world, much greater axial tilt (obliquity), an eccentricity equivalent to that of Mercury, receiving less radiation from its star, and more reflection from the surface. These parameters were tweaked from initial tests to result in something similar to the Earth. The final global mean temperature was 15.2616, which is closer to historical global mean of the real Earth than the Energy Balance Model predicts (18.1656). A graph of the global mean for both the Earth and the model exoplanet is shown on the same graph in Figure 12 along with northern hemisphere variability for both models. One can see that the exoplanet model here has less variability (about 8 degrees annually) compared with Earth (about 11 degrees annually).

Table 6: Parameter Settings for Energy Balance Models for Earth vs. Theoretical Habitable Exoplanet

Parameters	Mask	Initial Temp	Obliquity	Eccentricity	Solar Constant	Longwave a	Longwave b
Earth	Earth	20.0	23.45	0.017	1360.0	203.3	2.094
Exoplanet	Ocean	15.0	75.0	0.2056	1190.3	250.0	3.0

4 Discussion and Conclusion

As terrestrial-size exoplanets continue to be discovered, it will continue to be essential to understand more about what makes a planet habitable and suitable for life like our own. It is understandable to initially focus on properties that are the most like the Earth, but as our example demonstrated, we need not confine ourselves to such high levels of similarity. While it would of course be important to extend such models with more realistically complex climate models, the present examination makes it clear that such work would indeed be fruitful and necessary. There is so much that is not well-understood about the so-called Goldilocks Zone (or indeed, if such a thing is even meaningful). Our exploration of the Energy Balance model shows that even the Earth experienced sufficiently different conditions in the past, and will in the future, that confining ourselves to close similarity to the present Earth is likely to overlook many promising candidates. Pushing more complex models past tipping points would also help illuminate our present climate, as well as do much to advance our understanding of the models generally, and exoplanets in particular.

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