Hands-on Math Modeling for STEM and Statistics

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Three Examples

- **Laser Diffraction** – Algebra through Multivariable, and Statistics
- **Springs** – Trigonometry through Differential Equations
- **Dice** – Statistics and Probability

Laser Diffraction

Algebra & Statistics

- Unit conversions (within metric, and Standard to metric), scientific notation
- Generate a proportional model, including inverse proportionality, using multiple variables
- Models real world data students collect
- Linear and (intrinsically linear) power regression

Multivariable Calculus

- Multivariable functions
- Linear approximations
- Polar coordinates
- Spherical coordinates
- Physics applications connect with their other classes

- Yards sticks, tape measures (or other measuring device)

- A wall or screen
- Masking tape or other removable adhesive
- Marker or writing utensil

Warning!

Laser light can damage the eyes, even in reflection. Please use shaded safety glasses if staring at reflections for long periods, and DO NOT shine light into anyone's eyes.

You will need measuring tools (yardsticks), diffraction gratings (at least two sizes), lasers (at least two colours).

C. Experiment with the equipment and determine which variables seem to influence the pattern (the distance between bright spots and where the laser is shone) that you get on the wall. Were there any variables that did NOT affect the distance from the center to the spots? Write the variables below. Feel free to experiment with the materials available to you.

Variables that affect distance between bright spots	***Variables that do NOT affect distance between bright spots***

D. Do you think that the color of the laser would affect the distance between the center maximum and other maxima (bright spots)? Why? Use a second laser colour to check your conclusion.

Check with your instructor

Give students a chance to experiment and figure out what matters and what doesn't.

	Question	*Brief Description of Experiment*	*Independent variable & units*	*Dependent variable & units*	*Controls & units*
Experiment 1	If you change ______________ (independent variable) how does the distance between bright spots (dependent variable) change?	The ____________________________ (independent variable) will be varied to see how the __________________________________ (dependent variable) changes. The ____________________and ______________________ will stay the same.			
	If you change ______________ how does the ______________________ ______________________	The ____________________________ will be varied to see how the __________________________________			

Students design experiments they can test, with control variables.

Experiment 1 *How does ________________________________ depend on ___________________________________?*	
Controls and Units ____	
IV _____________________________________ ***Units*** _____________________________________	***DV*** _____________________________________ ***Units*** _____________________________________

Students collect data and then model it graphically and using regression functions in the calculator.

Experiment 1 tested the affect that changing the ______________________________had on the

______________________________________, while controlling the _________________________________

and _____________________________________.

Regression equation type	Regression equation obtained	r^2 or R^2 (4 decimal places)

A variety of regression equations can be tested to determine which is best based on a variety of factors, like correlation, or long-run behavior.

Springs

Trigonometry

- Sinusoidal functions
- Finding Amplitude
- Finding Phase Shifts
- Finding Horizontal Shifts
- Finding Period and Frequency
- Writing equations
- Translations
- Sine Regression

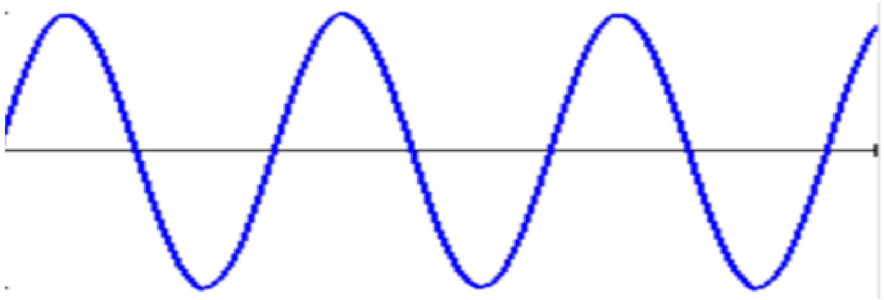
Differential Equations

- Exponential Decay
- Matching Real-world to Equations
- Springs with Mass (not massless)
- Modeling Velocity/Acceleration
- Connecting ideas across courses
- Problems of data collection
- Systems of Springs

More Than One Technology Works

- I use a motion detector to plot the oscillations of the springs, with index cards on the bottoms of the springs (or mass cups) so that the motion detectors can "see" the spring.
- You can do this experiment with a meter stick and a cell-phone video camera. Letting time be in frames and using the frames to determine maximum and minimum heights, equilibrium, etc. These points can be measured and put into the calculator manually.
- Kick this up a notch to talk about superposition with systems of two or three springs.

Q. The graph of a sinusoidal "wave" is shown below. Label key points on this graph with your period, frequency, amplitude and equilibrium from your data.



R. Assuming the amplitude remains constant over time, we can write the equation of the position of the mass as $y(t) = A\sin(\omega t + \delta) + C$. Store this equation in Y1. Confirm your results by using the Sine Regression function in your calculator as follows:

Press STAT, then scroll over to CALC, and scroll down to C: SinReg. The newer TI-84s will show a screen like the one shown here. Up the iterations to 10. Select the lists where your data is stored (X=time). Fill in the period you found in M. Store the regression equation in Y2 by selecting VARS, and then Y-VARS, FUNCTION, and Y2. Finally, select Calculate.

```
SinReg
Iterations:3
Xlist:L1
Ylist:L2
Period:█
Store RegEQ:
Calculate
```

On older calculators without the Stats Wizard, your screen will look like the last line shown. Specify the lists and the function to store the equation if they are not the defaults L_1 and L_2. You should see output that looks similar to this:

```
SinReg L1,L2,Y1
```

```
SinReg
y=a*sin(bx+c)+d
a=19.45252523
b=.1707079588
c=-2.024072777
d=19.85520973
```

Compare your results to those obtained from the calculator. Graph both functions. Which one appears to be a better fit? Why?

Students are asked a series of questions to tease out features of the graph, and compare to calculator solutions. Trig students can stop here.

V. To model this function, use one of the regression functions under STAT, CALC. Which model fits the data the best? Record your sample models and r^2 values here.

Regression model type	Regression Equation	$r^2 or R^2$	Does the equation fit properties of the data (particularly end behavior)?

More advanced students can be asked to model the decaying amplitudes. And the relationship between mass and frequency. The non-zero mass of the spring matters here.

KK. Solve the differential equation $my'' + \gamma y' + ky = 0$ symbolically. How does the frequency depend on mass in your solution? Does it agree with your answers in W and EE/FF?

Check with your instructor.

LL. Can you determine the spring constant and damping constant of your spring using the information obtained from your equations? [Hint: how many variables are there to solve for, and how many equations for them can we determine from our two models?]

DiffEq students often struggle with springs, but they can relate the equations back to their models.

Dice
Statistics

- Data Collection
- Simulations
- Linear Regression & Power Regression
- Probability and Law of Large Numbers
- Central Limit Theorem
- Interpreting Equations
- Technology

Tetrahedral (D4, four-sided) dice

Standard cubic (D6, six-sided) dice

Octohedral (D8, eight-sided) dice

Decahedral (D10, ten-sided) dice; come in two varieties

Dodecahedral (D12, twelve-sided) dice

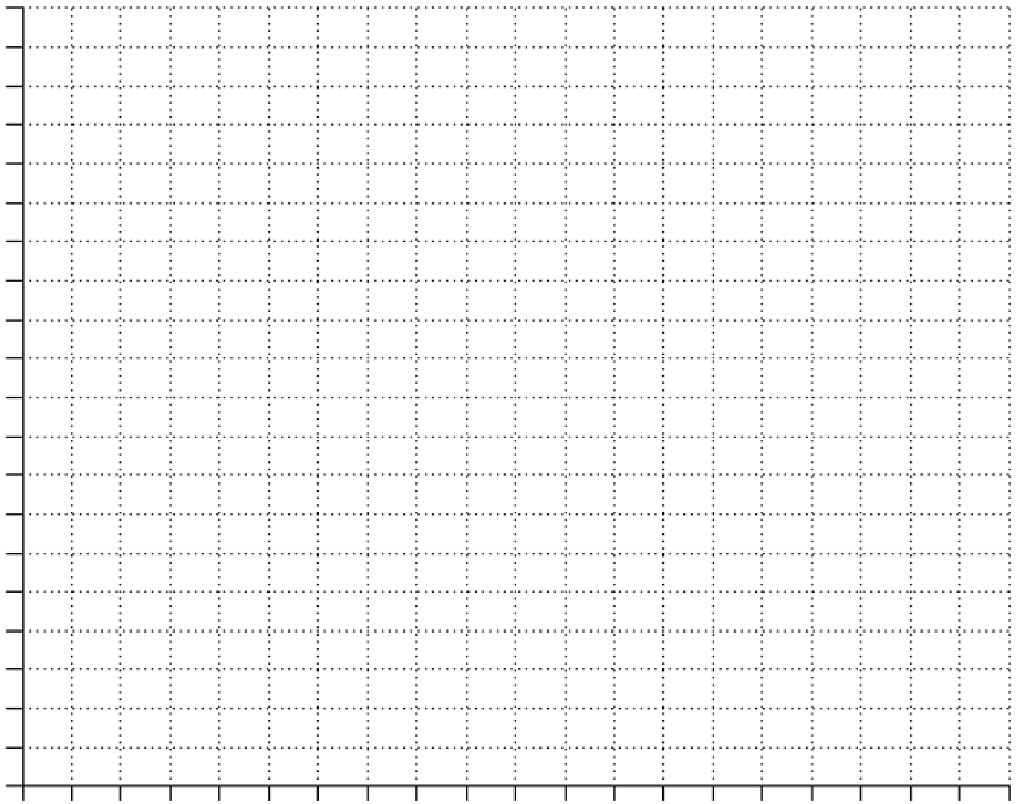
Icosahedral (D20, twenty-sided) dice

Tetraicosahdehron (D24, twenty-four-sided) dice

Tricontahedron (D30, thirty-sided) dice

Kick up your dice experiment with gaming dice. Encourage your students to choose something more interesting than evens and odds.

F. Enter the data into your calculator with the first column (total number of dice) as L_1, and the second column (# of successes) as L_2. Use the calculator to create a scatterplot and reproduce the graph below. Label the axes appropriately.



Students create a scatter plot, and model first with two points, and then with regression.

```
MATH NUM CMPLX PROB FRAC
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

The syntax for this function is **randBin(# of trials, probability of success)**. You can add a third piece to the syntax for number of repetitions. Newer calculators will give you a "wizard" screen to remind you what each piece means.

Enter the number of trials, the proportion you found in C, and since we want 5 repetitions of each sample size, enter 5 as the last value. Then select paste. Your screen will appear as shown below.

```
randBin
n:500
p:.5
repetitions:5
Paste
```

```
randBin(500,.5,5)
    {258 258 240 258 236}
```

The numbers shown are the number

S. We don't have that many dice to roll physically (nor would we really want to), but we can use technology to simulate the roll of many dice using a random number generator.

Access the website random.org. Scroll down to the *Numbers* section and select *Integer Generator*.

You will want several large sets of numbers (for instance: 200, 500, 1000, 10 000 each at least once, but preferably, 5 times each). However, counting these numbers by hand is still tedious, so just do this for a sample size of 200. You can select the number of columns to print the results in. If you choose 10 columns, each one will have 20 numbers each, which will be manageable to count. While you can do this on a computer screen, it may be beneficial to print the 5 lists of numbers in order to facilitate counting. Set your integer range to match the values on your dice. An example for an icosahedral dice is shown in the image above for a sample size of 100. Set the number of random outcomes you want, and then select the Get Numbers button

Simulations can kick up the sample sizes using online tools or the calculator. Introduce students to binomial variables.

Sample Size	Sample Proportion #1	Sample Proportion #2	Sample Proportion #3	Sample Proportion #4	Sample Proportion #5

Converting successes to proportions opens a discussion about 0 correlation and horizontal lines, and the Law of Large Numbers.

column of table with the data using the sample size (number of total dice tossed), and the estimate of the standard deviation at that sample size.

AA. What do you notice about the standard deviation estimates as sample size increases?

BB. Clear out the old data from your calculator (or use a different pair of lists) and enter the data in the table.

When that this done, go to STAT, then CALC menu and then scroll down to **A: PwrReg**. This function gives a regression equation in the form $y = ax^b$. Recall that is b is close to one, this is a linear equation with a zero intercept. If b is close to two, then this is a parabolic graph. If b is close to ½, this is a square root function. If b is negative, the power is in the denominator.

CC. What equation did you get? What is the correlation?

# of dice rolled	Proportion Standard deviation (estimate)	Proportion Standard deviation calculation (true)

Taking multiple samples of the same size let's us calculate a standard deviation of a sampling distribution and explore the Central Limit Theorem for proportions.

Time Commitment

- Typical students can generally complete 3-4 pages of these projects in an hour of class time. This time can be used to collect data, work on analyses, etc.
- Students can complete some of the questions outside of class, but for projects that occur over several days, review their answers with them in groups before they move on to new material.
- Coupled with traditional lecture, these projects can be a valuable look ahead to new material coming up (and become a common reference point for the class), or can be done as a final project to pull ideas together already discussed.

Cost

- Gaming dice can be purchased by the pound for about $17 in a variety of sizes.
- Laser pointers are much less expensive than in the past: $5-6 for red, $15-20 for blue or green. (Yellow is still ~$300.)
- Diffraction gratings run from a couple bucks each to $30.
- Motion detectors run around $170 but they are sometimes given away as premiums for compatible calculator purchases, and there are less expensive alternatives (all the kids have phone cameras).

Acknowledgements

- The Lasers and Springs projects have been adapted from modeling projects developed by Beth Basista (Wright State University) and Rodney Null (Rhodes State Community College) through an NSF ATE grant. That grant supported a modeling workshop that I attended in Ohio in 2015.
- The Dice project was developed by me as part of that workshop.

LET'S PLAY!

Project Files

- Laser-Diffraction Project -- http://betsymccall.net/prof/courses/spring16/aacc/Lasers_Diffraction_Grating_Project.pdf
- Spring Project -- http://betsymccall.net/prof/courses/spring16/aacc/Modeling%20Springs.pdf
- Monte Carlo (Dice) Project -- http://betsymccall.net/prof/courses/spring16/aacc/Monte%20Carlo%20Modeling.pdf

- For editable Word versions of these projects, you can email me at bjmccall@aacc.edu and I can send them to you.